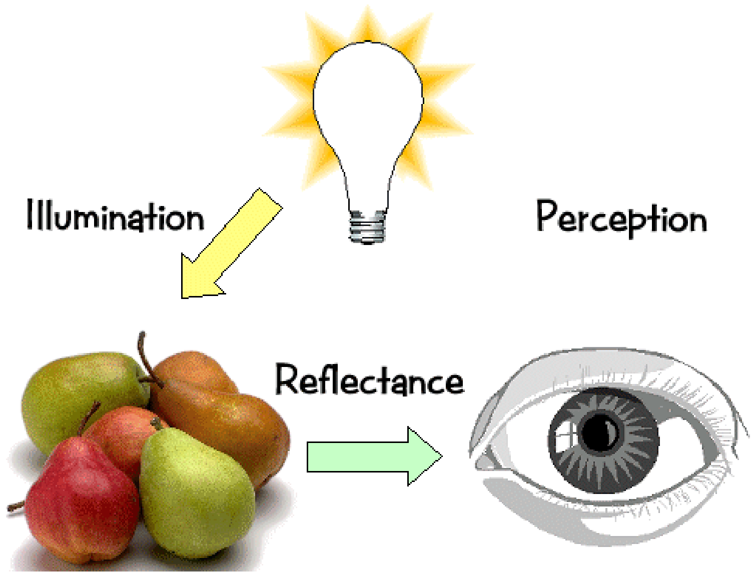
3. Reflectance & Lighting

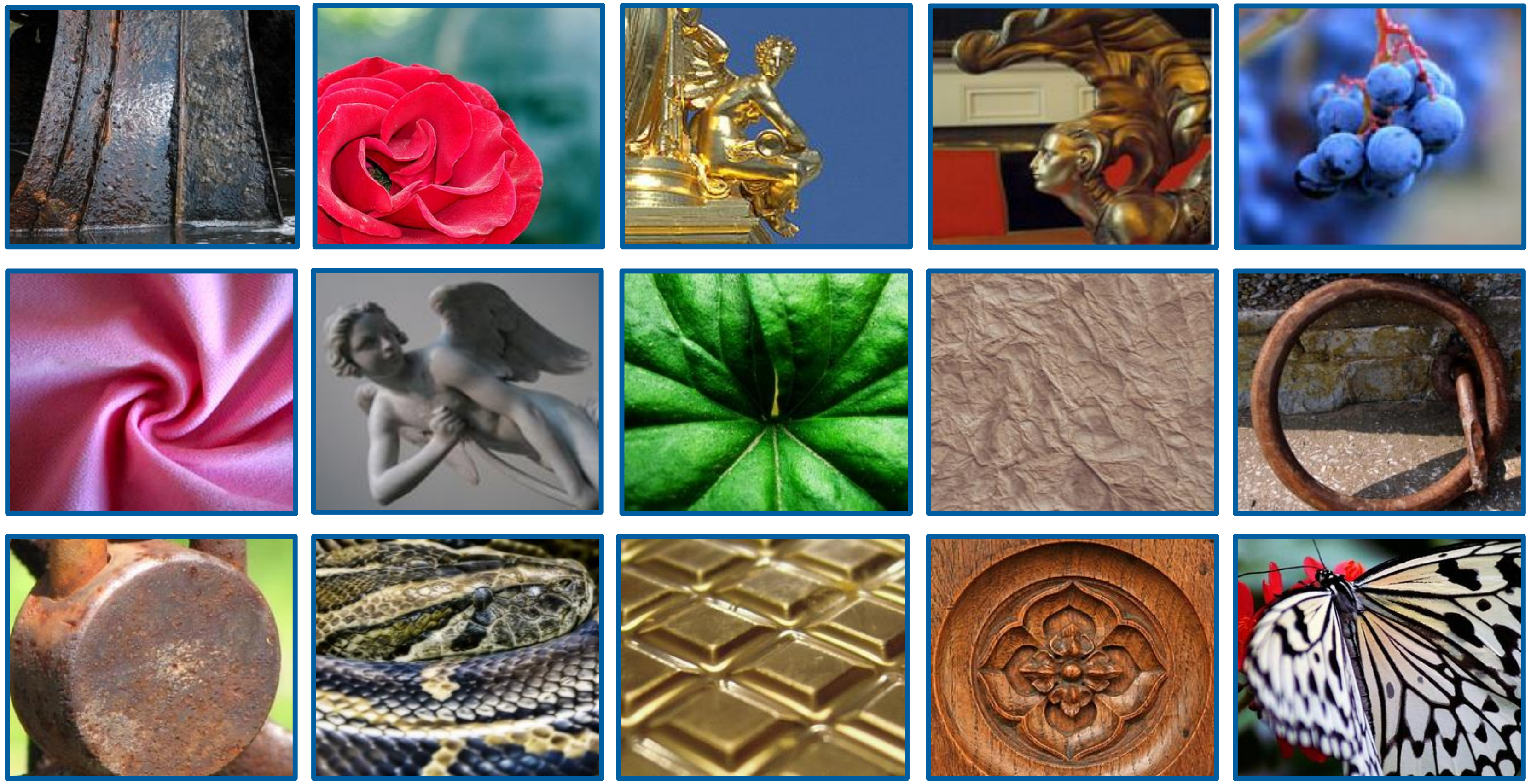
Lights, Material/Reflectance, and Geometry

- The radiance at a pixel is determined by:
  - Lighting (direction & intensity)
  - Material/reflectance
  - Local shape (mainly surface normal direction)



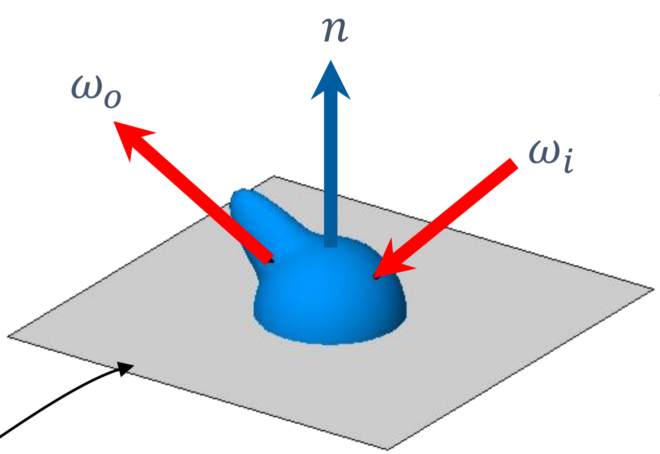
Light at Surfaces

- Many effects when light strikes a surface -- could be:
  - Reflected, refracted, scattered, absorbed, etc.
  - We focus on reflectance today for simplicity



Material/Reflectance

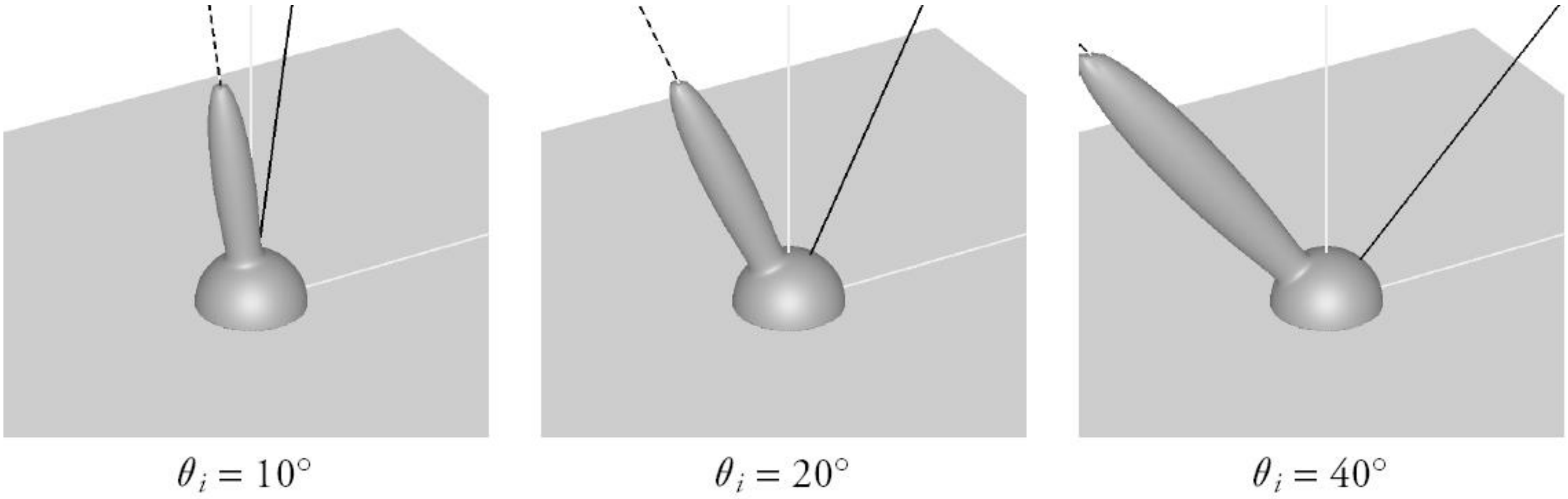
- *Reflectance* is all about the way light interacts with surfaces
- It is an entire field of study on its own
- The most important quantity is the BRDF (Bi-directional Reflectance Distribution Function)
- A BRDF $\rho(\omega_o, \omega_i)$ is a function of two directions
  - Incident lighting direction ω_i
  - Reflected lighting direction ω_o



Both directions are defined in a local coordinate system where typically the surface normal direction n is the z-axis

The BRDF

- It describes how reflected lights are distributed
- It is a pdf function for each fixed ω_i
  (the distribution of reflected energy)
- This distribution changes when the incoming ray changes



visualize a BRDF as a function of ω_o for a fixed ω_i;
the radius along each direction is set to the radiance of the reflected light at that direction.

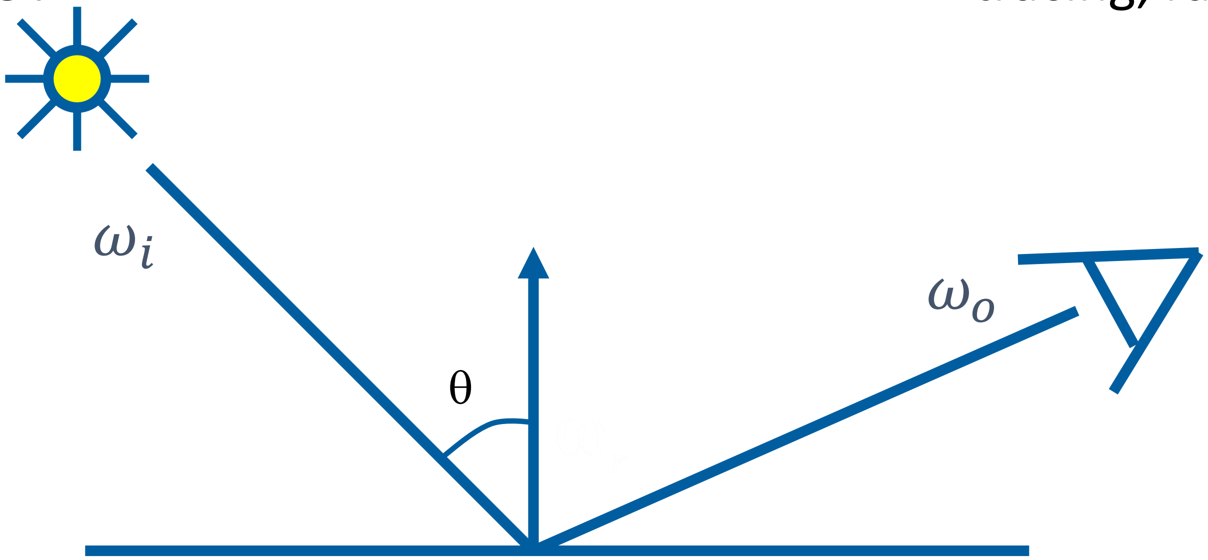
Local Assumption

- BRDFs assume reflectance is local: all light leaving a point depends ONLY on the light arriving at that point
- It ignores many non-local behavior, e.g.
  - Translucency: semi-transparent materials, e.g. marble, human skin, etc
  - Fluoresce: absorbing lighting in one wavelength and emit in a different wavelength
- In this class, we further ignore non-local effects, e.g.
  - Inter-reflection
  - Cast shadow

The Rendering Equation

- Why study BRDFs?

The basis for computer graphics rendering (e.g. ray tracing, radiosity, etc)



$$L_o(\omega_o) = \rho_{bd}(\omega_o, \omega_i) L_i(\omega_i) cos\ \theta_i$$

Reflected Radiance (Pixel Intensity)

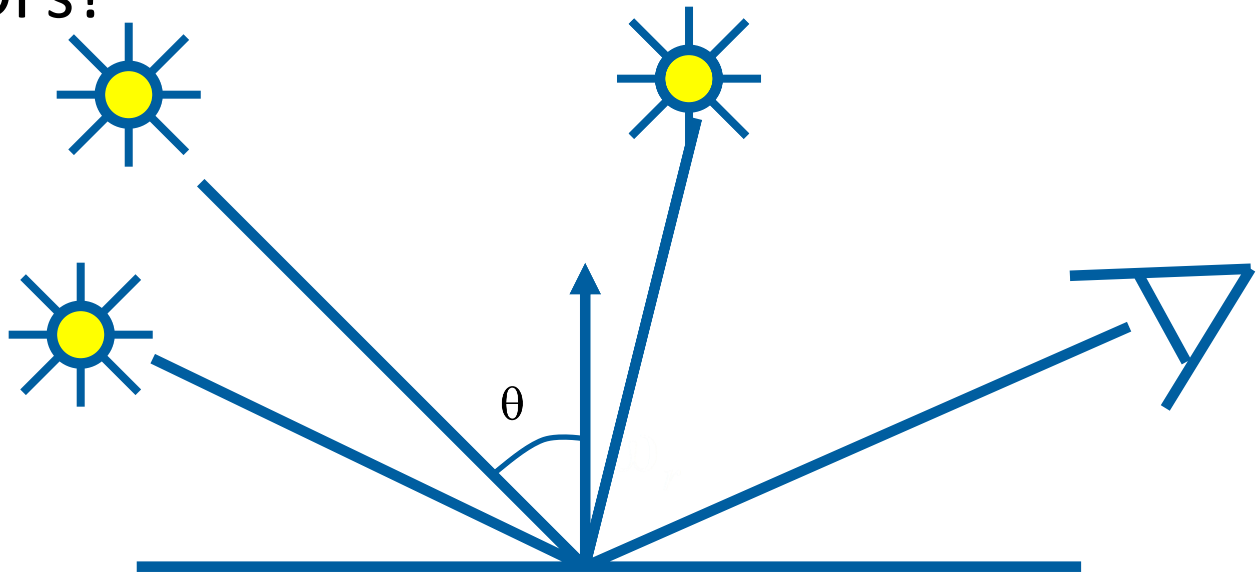
BRDF

Incident radiance (from light source)

Cosine of Incident angle

The Rendering Equation

- Why study BRDFs?



Sum over all light sources

$$L_o(\omega_o) = \sum_i \rho_{bd}(\omega_o, \omega_i) L_i(\omega_i) cos\, \theta_i$$

Reflected Radiance (Pixel Intensity)

BRDF

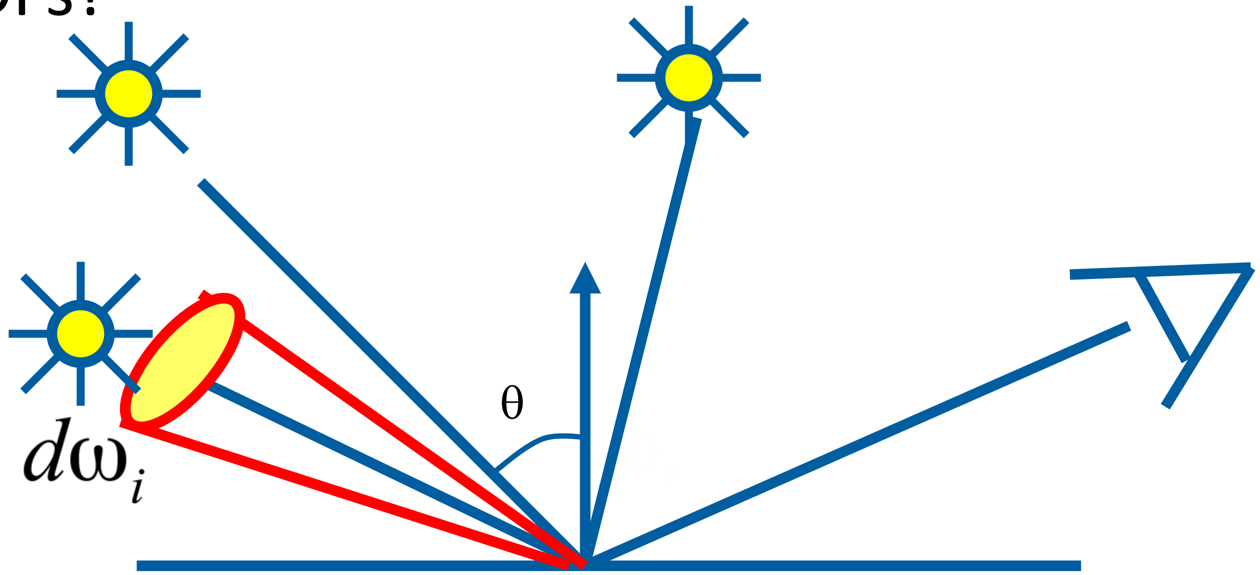
Incident radiance (from light source)

Cosine of Incident angle

The Rendering Equation

The rendering equation consider all points in a scene. What we see here is only one point.

- Why study BRDFs?



Replace sum with integral

$$L_o(\omega_o) = \int_{\Omega} \rho_{bd}(\omega_o, \omega_i) L_i(\omega_i) cos\ \theta_i\, d\omega_i$$

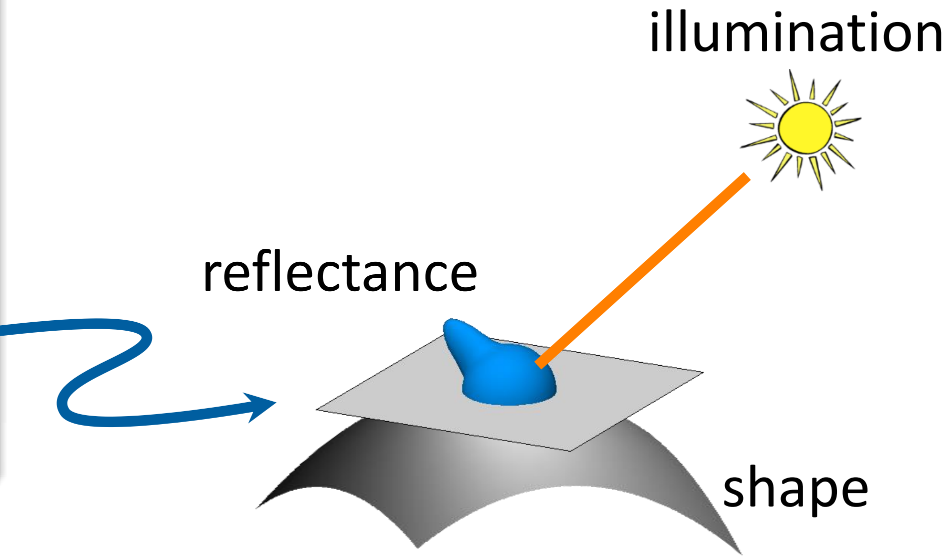
Reflected Radiance (Pixel Intensity)

BRDF

Incident radiance (from light source)

Cosine of Incident angle

Radiometric Image Analysis

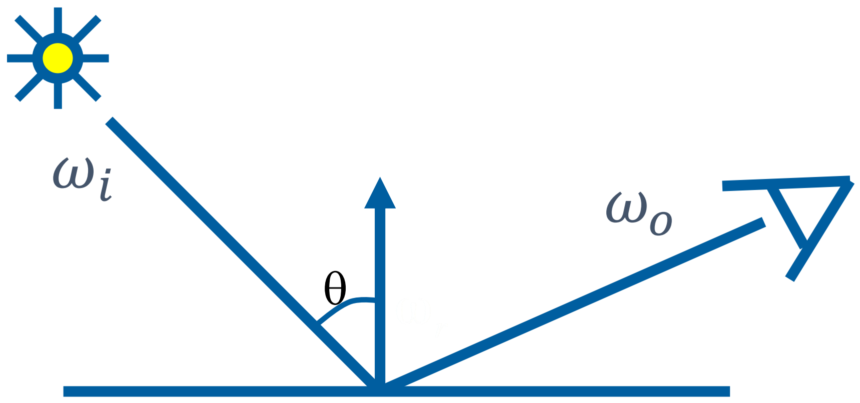


- The rendering equation:
  determine irradiance (pixel values) from shape, lighting, and reflectance
- Radiometric image analysis:
  recover shape, lighting, or reflectance from irradiance (pixel values)

Radiometric Image Analysis

Typical simplification assumptions:

- Single point light source (simplify the light source)
- No inter-reflection, no cast-shadow (ignore global shape effects)
- Simplified BRDF models (simplify the reflectance)

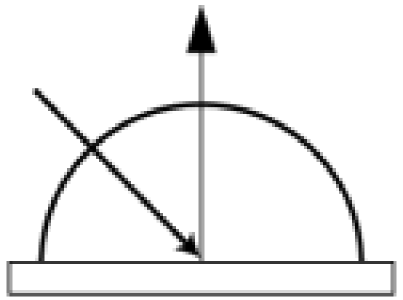


$$L_o(\omega_o) = \rho_{bd}(\omega_o, \omega_i) L_i(\omega_i) cos\ \theta_i$$

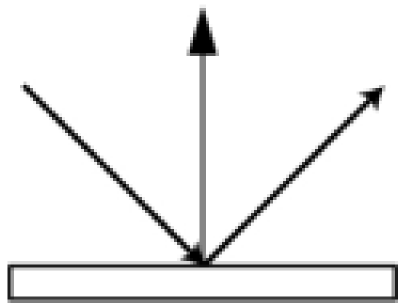
Questions?

Diffuse & Specular Reflection

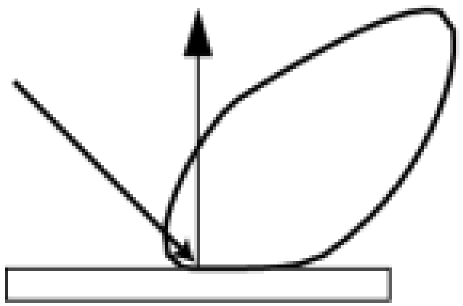
- Diffuse reflection:
  - The surface look the same from all directions (many vision algorithms depend on this!)
  - Matte surfaces
- Specular reflection:
  - The surface look different from different directions (causes troubles to many vision algorithms)
  - Shiny surfaces



ideal diffuse reflection (e.g. walls)



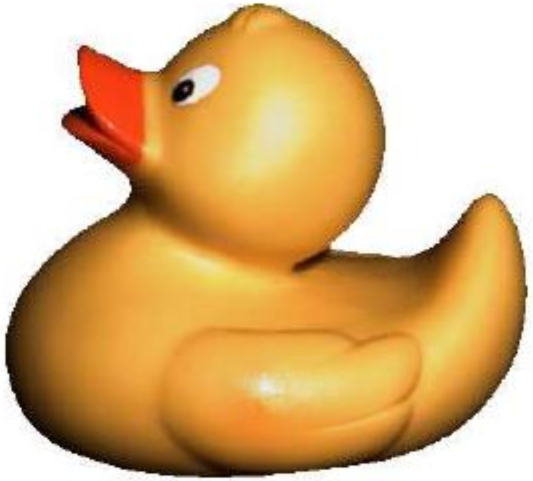
ideal specular reflection (e.g. mirror)



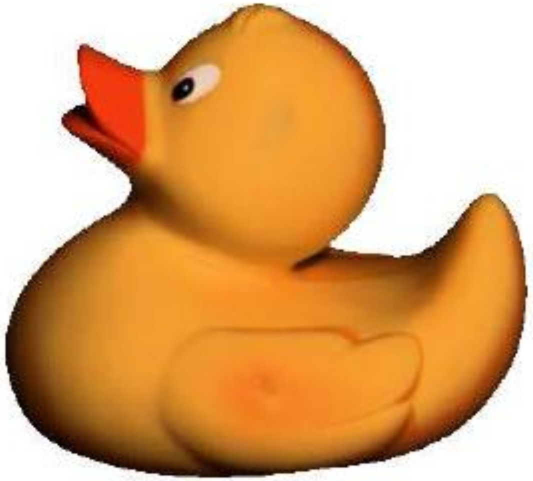
specular reflection (e.g. plastic, metal, porcelain)

Diffuse & Specular Reflection

- Diffuse reflection:
  - has the same color as the object surface
  - is unpolarized
- Specular reflection:
  - has the same color as the light source
  - has the same polarization as the light source



the original image



diffuse reflection

specular reflection

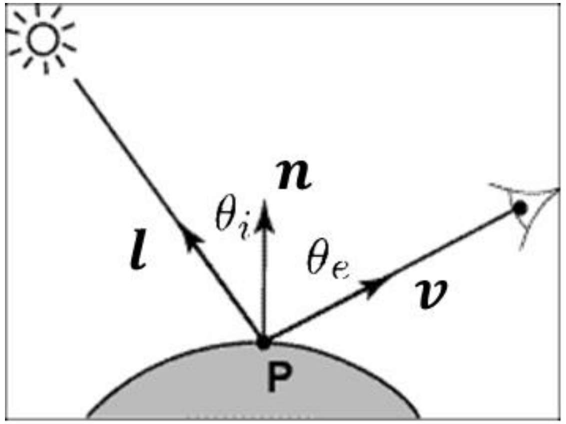
Lambert’s Model (Diffuse Reflection)

Johann H. Lambert

- Empirical mathematic model for diffuse reflection
  - Assume the BRDF is a constant $\rho(\omega_o, \omega_i) = \rho_0$
  - Observed Pixel intensity should be

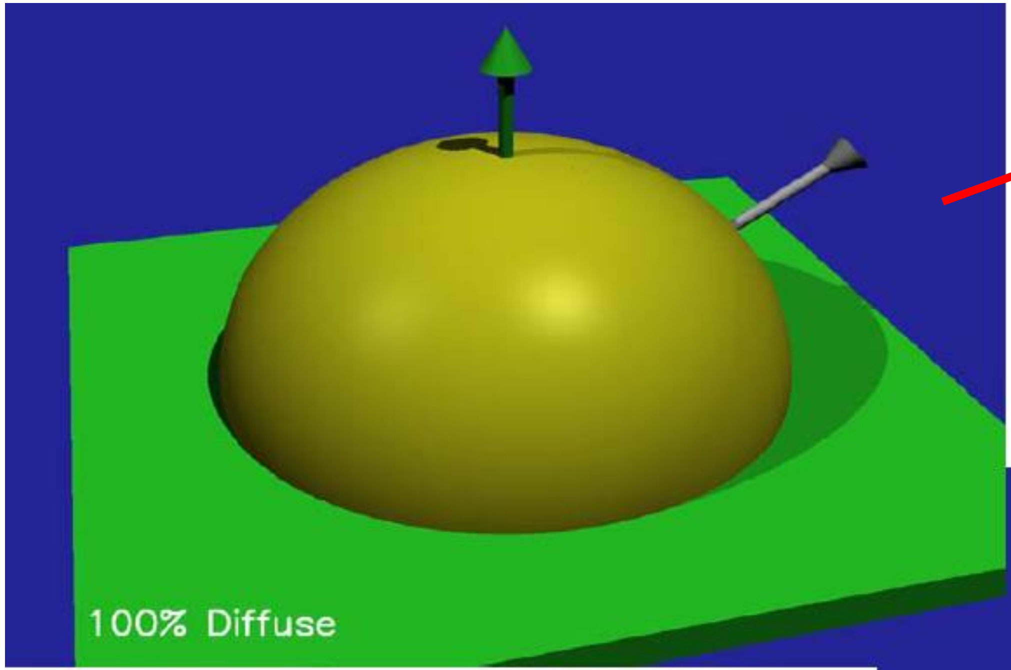
$$L_o = L_i \rho_0 \cos\theta_i = L_i \rho_0 \boldsymbol{n} \cdot \boldsymbol{l}$$

L_i and L_o are intensity of incoming and outgoing light



- Features of this model:
  - A pixel’s brightness does not depend on viewing direction
  - Brightness DOES depend on direction of illumination
  - This is the model most commonly used in computer vision (multi-view photo-consistency: the same 3D point look the same across views)

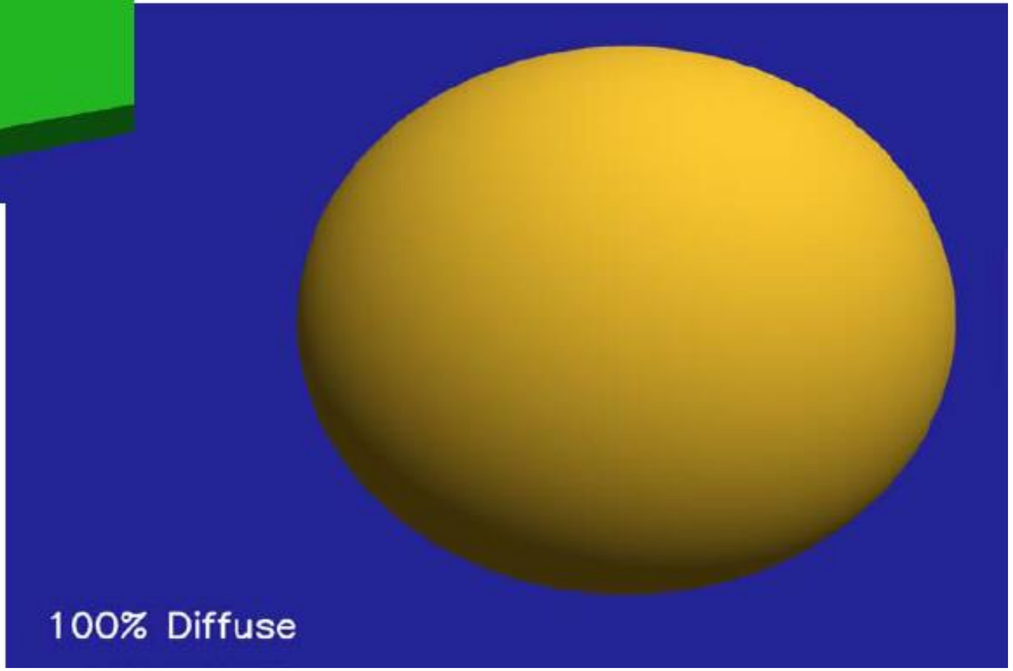
Lambert's Model



plot a BRDF as a function of ω_o for a fixed ω_i

3D plot of reflected intensity

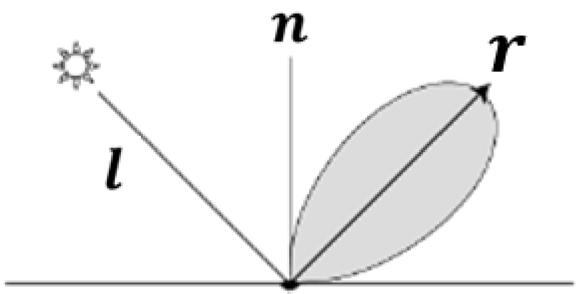
Appearance of a diffuse (dull) sphere



Phong Model (Specular Reflection)

- Mathematic model for specular reflection
  - Assume light is concentrated on the "mirrored direction" $\boldsymbol{r}$, $\boldsymbol{r} = 2(\boldsymbol{n} \cdot \boldsymbol{l})\,\boldsymbol{n} - \boldsymbol{l}$
  - Intensity of light falls off by cosine law
  - Observed Pixel intensity should be

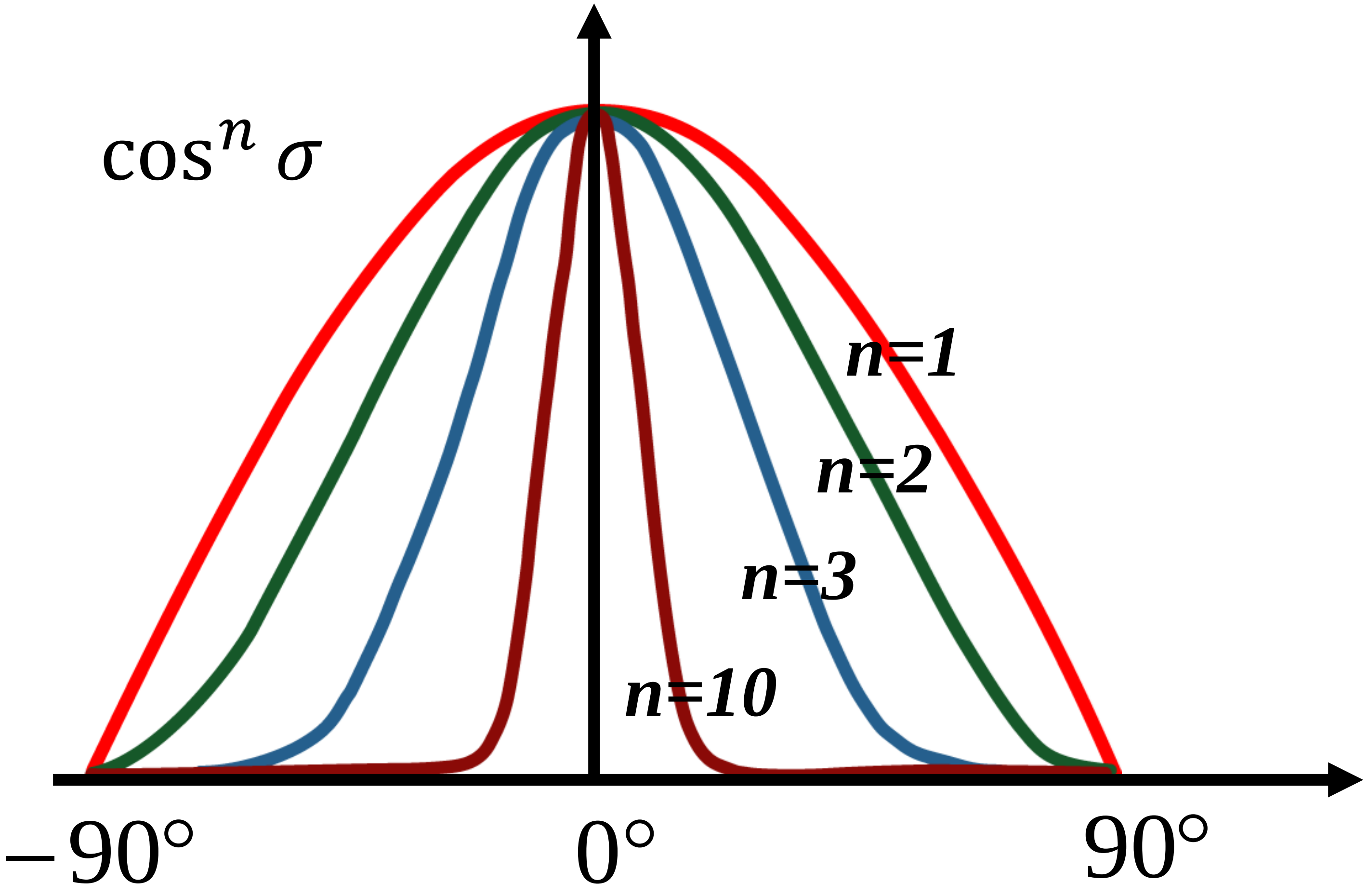
$$L_o = L_i(\boldsymbol{v} \cdot \boldsymbol{r})^n$$



- Features of this model:
  - A pixel's brightness depends on viewing direction
  - This is an empirical model, not physically correct!
    (e.g. violate energy conservation)

Phong Model

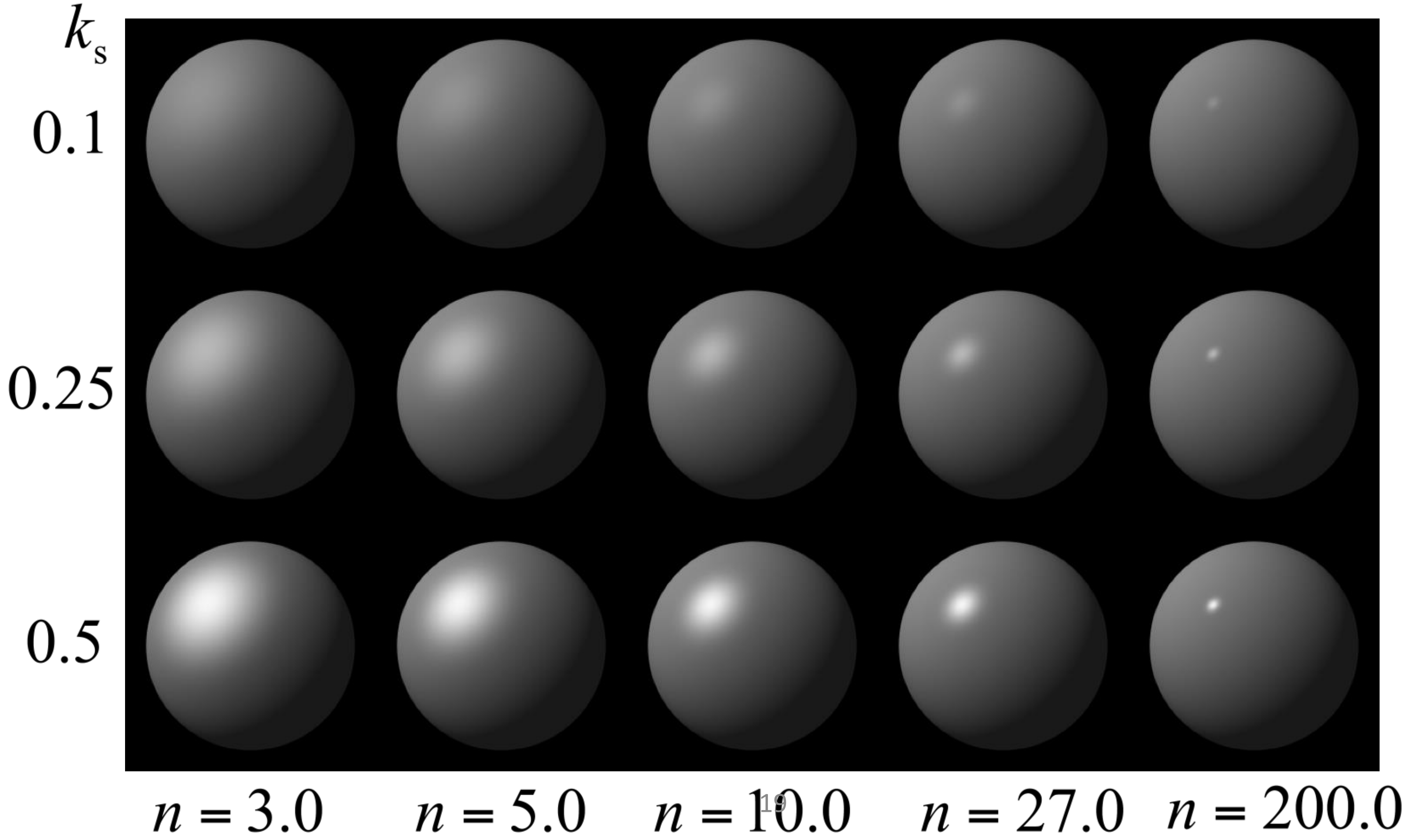
Shininess n controls the size of the highlight spot



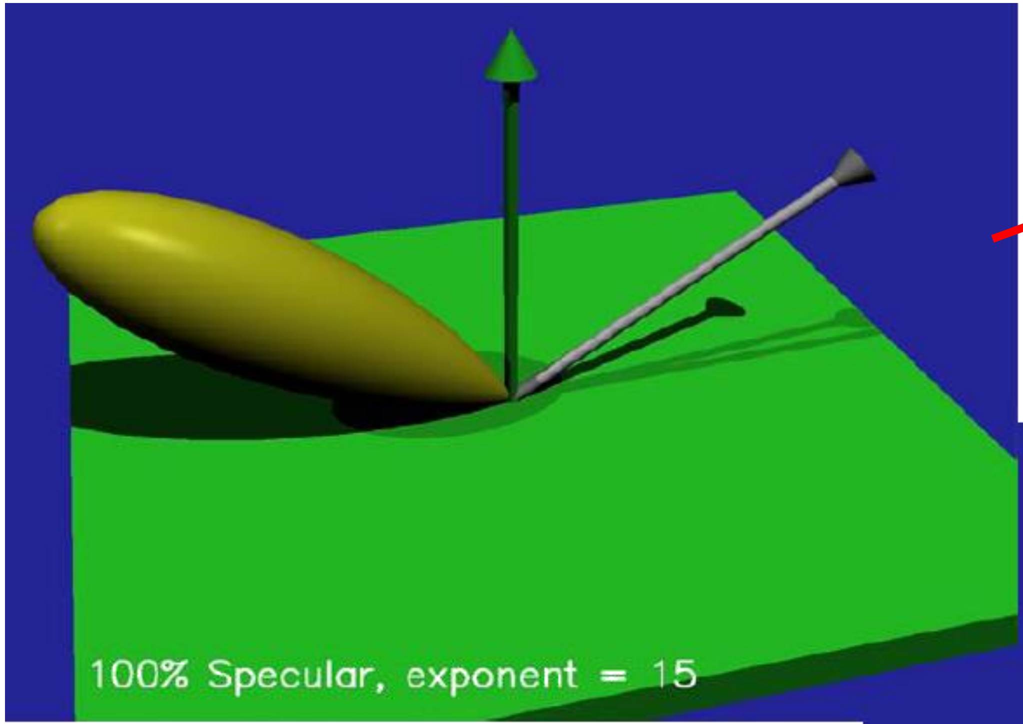
Phong Model

- Linear combination of Lambert's model and Phong Model

$$L_o = k_d \boldsymbol{n} \cdot \boldsymbol{l} + k_s (\boldsymbol{v} \cdot \boldsymbol{r})^n$$



Phong Model

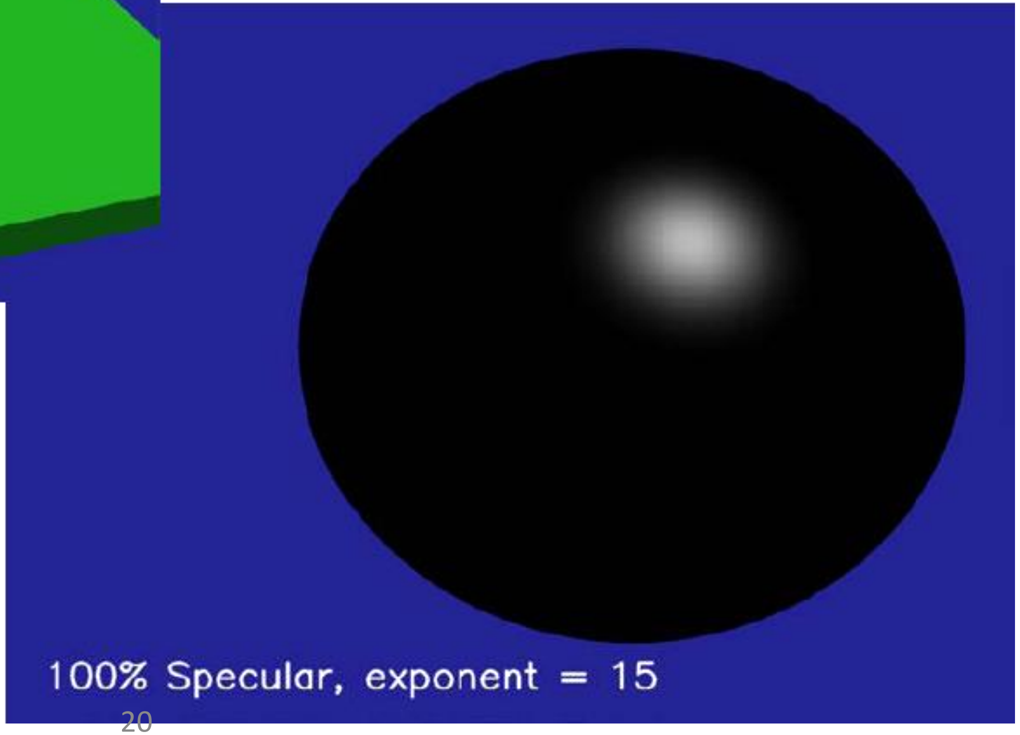


plot a BRDF as a function of ω_o for a fixed ω_i

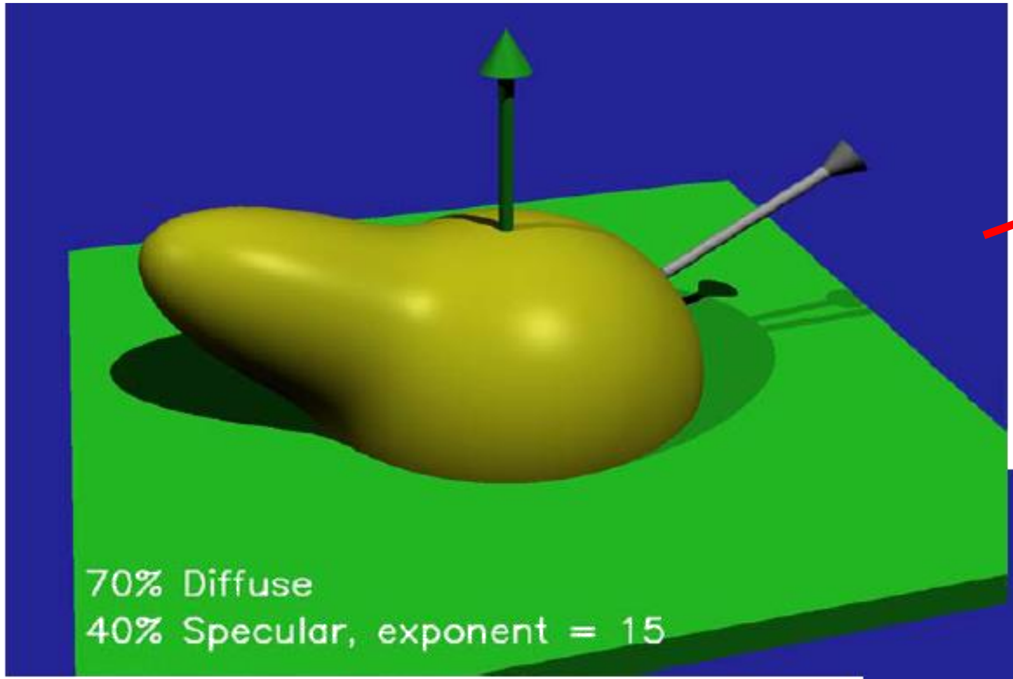
3D plot of reflected intensity

n = 15

Appearance of a specular sphere

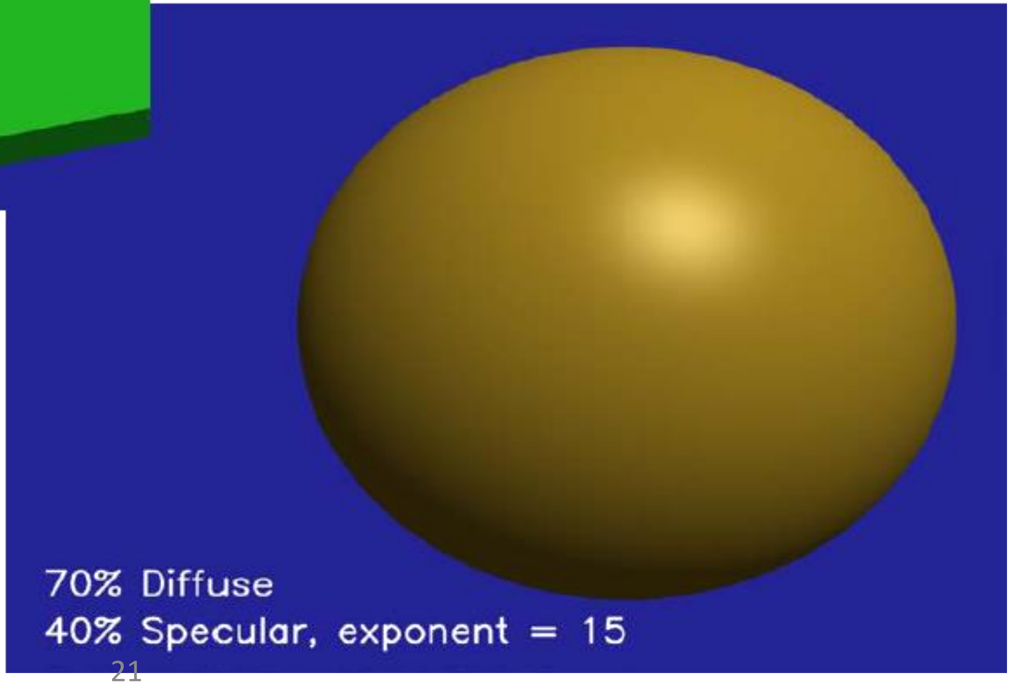


Phong Model



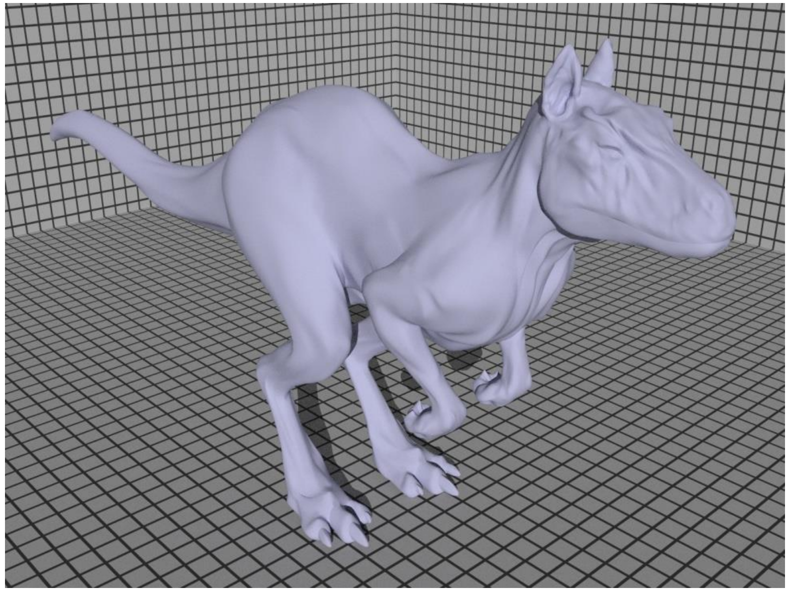
plot a BRDF as a function of ω_o for a fixed ω_i

$k_d = 0.7$
$k_s = 0.4$
$n = 15$

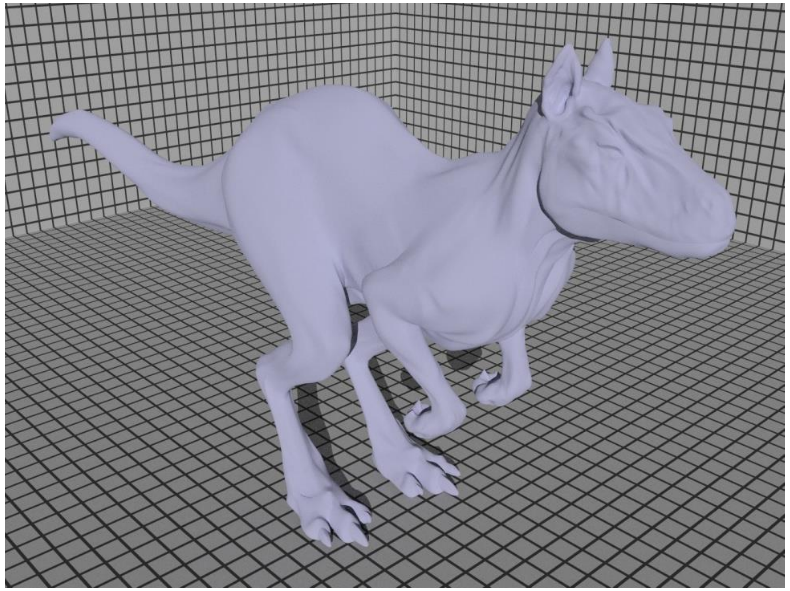


Many More Advanced Models

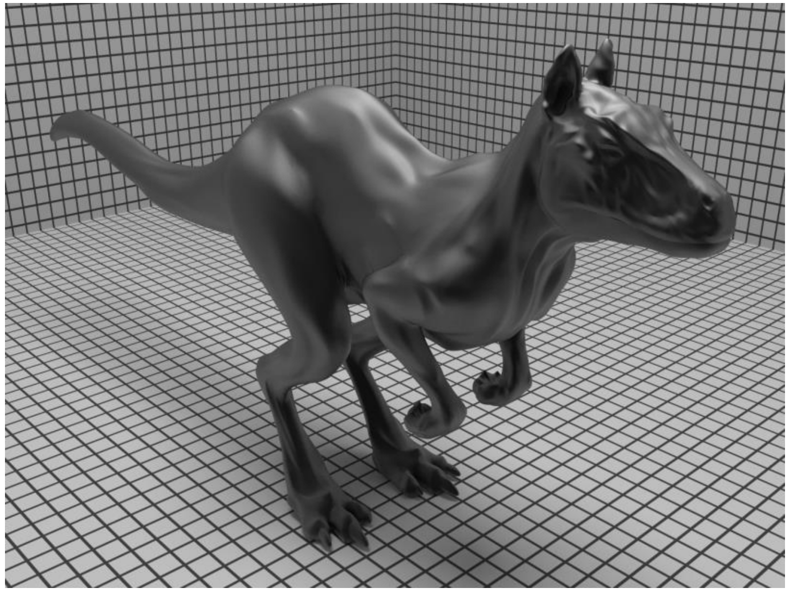
- To describe reflectance more faithfully
  - Oren-Nayar model (a diffuse reflectance model)
  - Cook-Torrance model (a specular reflectance model)
  - Ward's model (a specular reflectance model)



Lambert's model

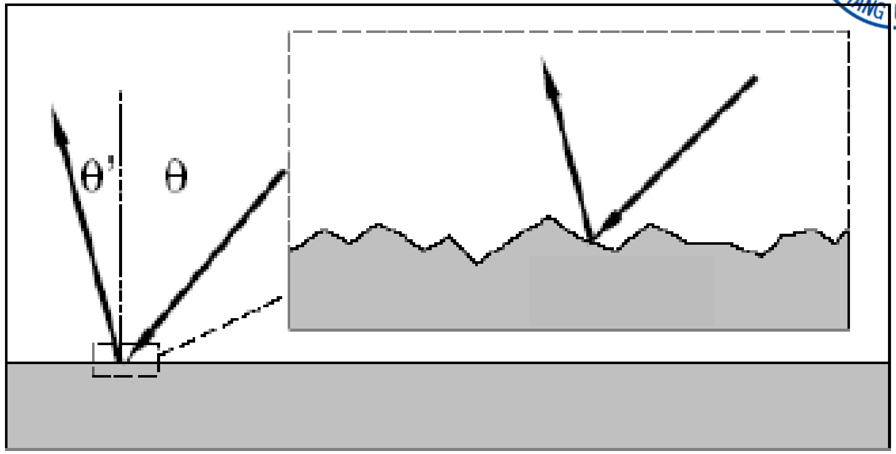


Oren-Nayar model



Cook-Torrance model

Microfacet Theory



- Assumptions:
  - The surface consists of microfacets at the microscopic level.
  - Facets are small enough (not visible) and big enough (no interference & diffraction)
- The aggregate behavior of these facets determines the reflectance.
- Two important factors:
  - How individual facet reflects light?

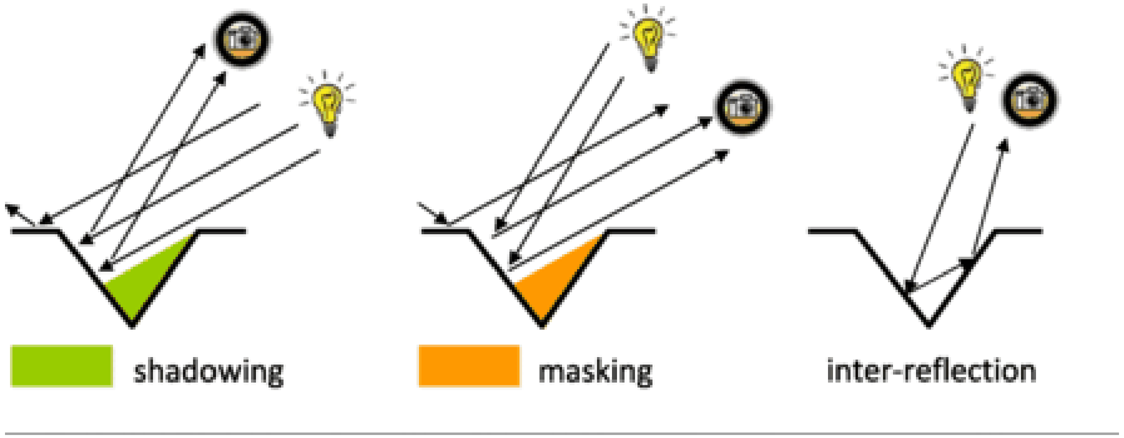
    e.g. perfect mirrors (Cook-Torrance) or perfect Lambertian (Oren-Nayar)

  - What is the distribution of facet orientations (normal distribution function)?

    e.g. Gaussian distribution

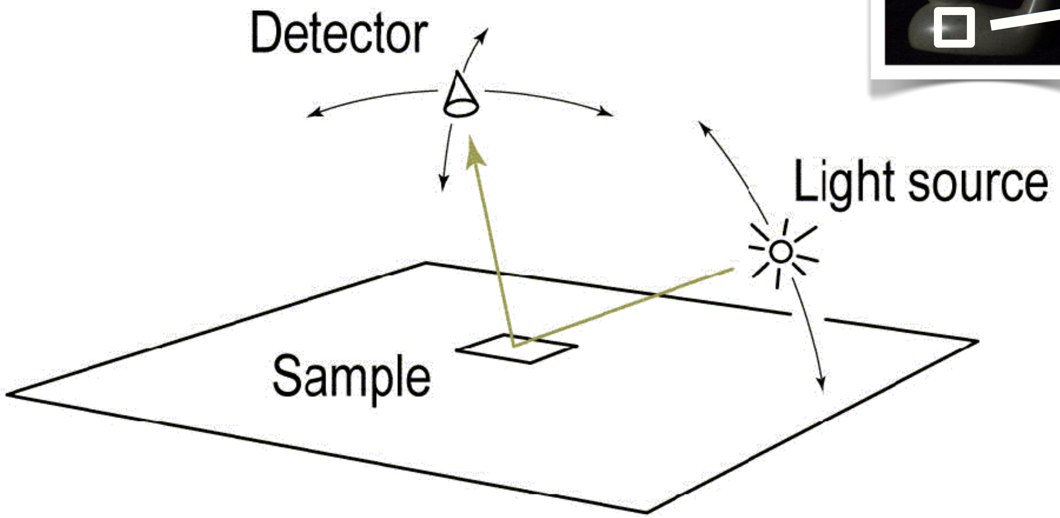
Microfacet Theory

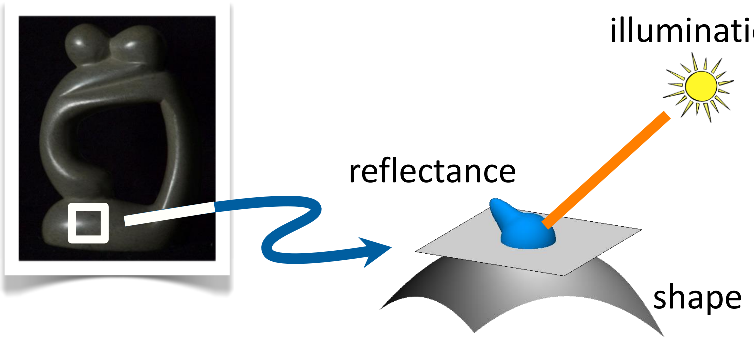
- Example: “V-grooves” on brushed metal surface



Questions?

Capturing BRDFs

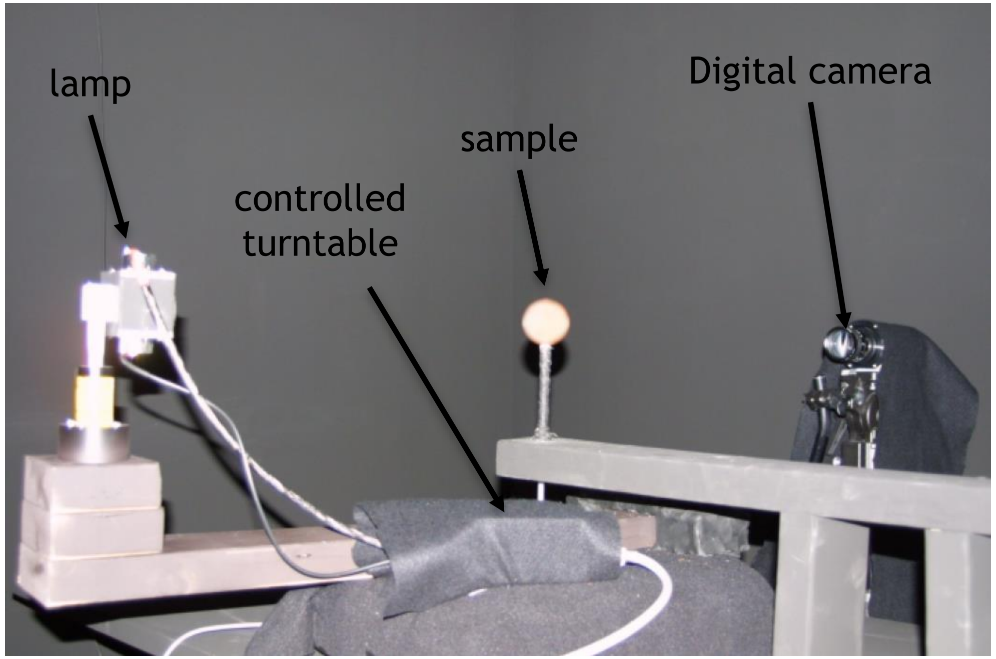


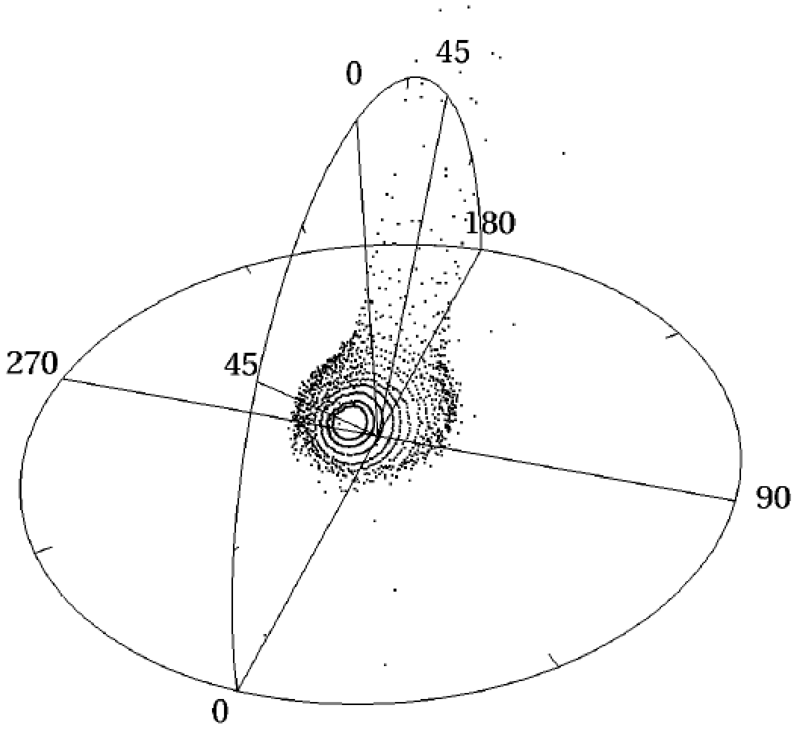


- Radiance (pixel intensity) is determined by shape, lighting, and BRDF
- Capture BRDF from images with known shape and lighting
- Often capture a flat sample with a moving light and camera
  - Need careful calibration of light and camera
    (also a darkroom to avoid inter-reflection, e.g. from the white walls)

More Efficient BRDF Capture

- Use a homogeneous spherical sample of the material
  - A sphere (known shape) contains all kinds of normals
  - So a single image contains many BRDF samples
  - Still need to move the light or camera



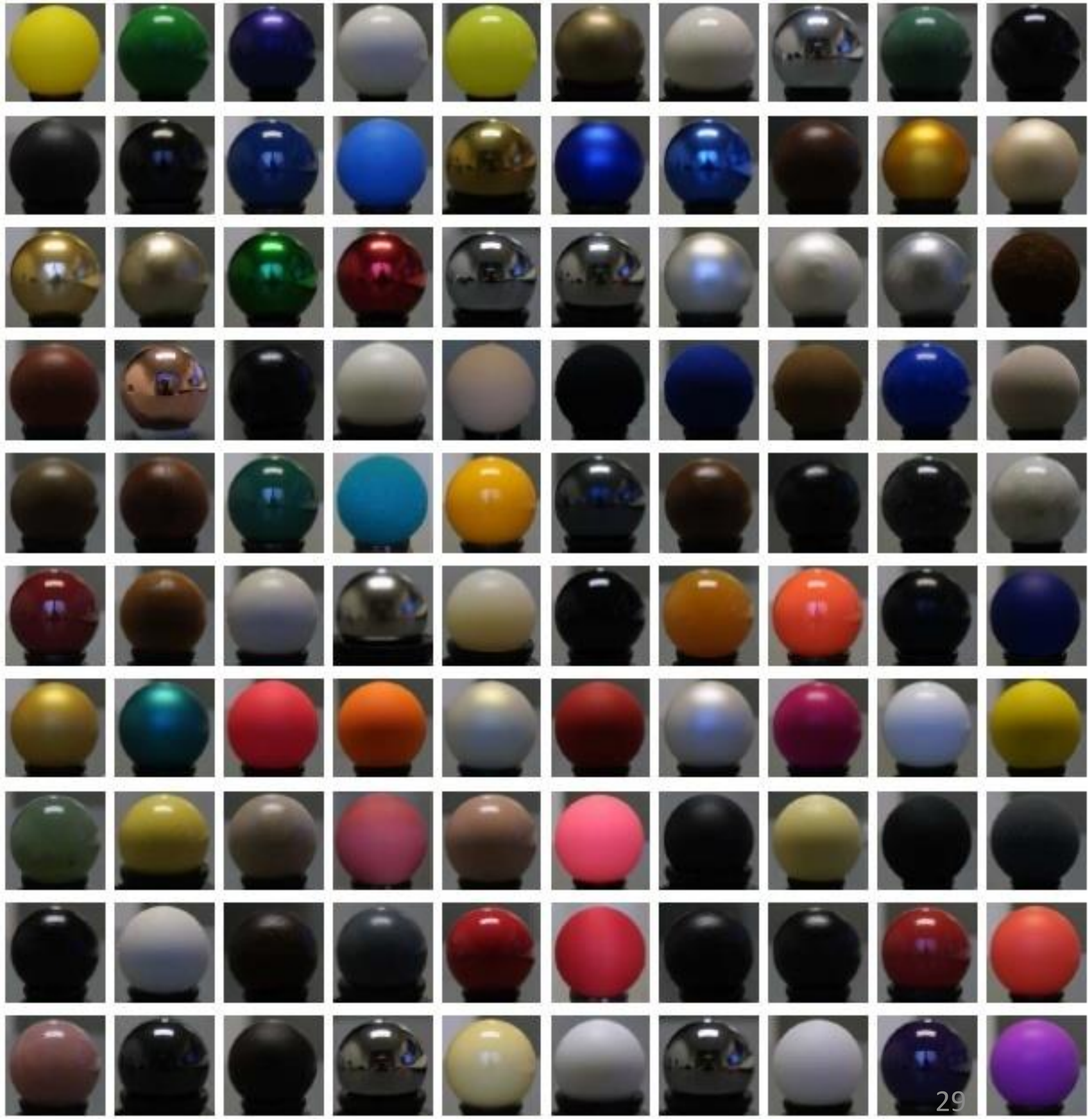


Represent Captured BRDFs

- Tabulated BRDF
  - 4D table $\rho(\omega_i, \omega_o) = \rho(\theta_i, \phi_i, \theta_o, \phi_o)$
  - Not editable

- Measure-then-fit analytic models
  - Fitting can reduce noise but also is limited by the model
  - Non-obvious error metric for fitting – often biased to specular which has large values
  - Difficult optimization – nonlinear; depends on initial guess

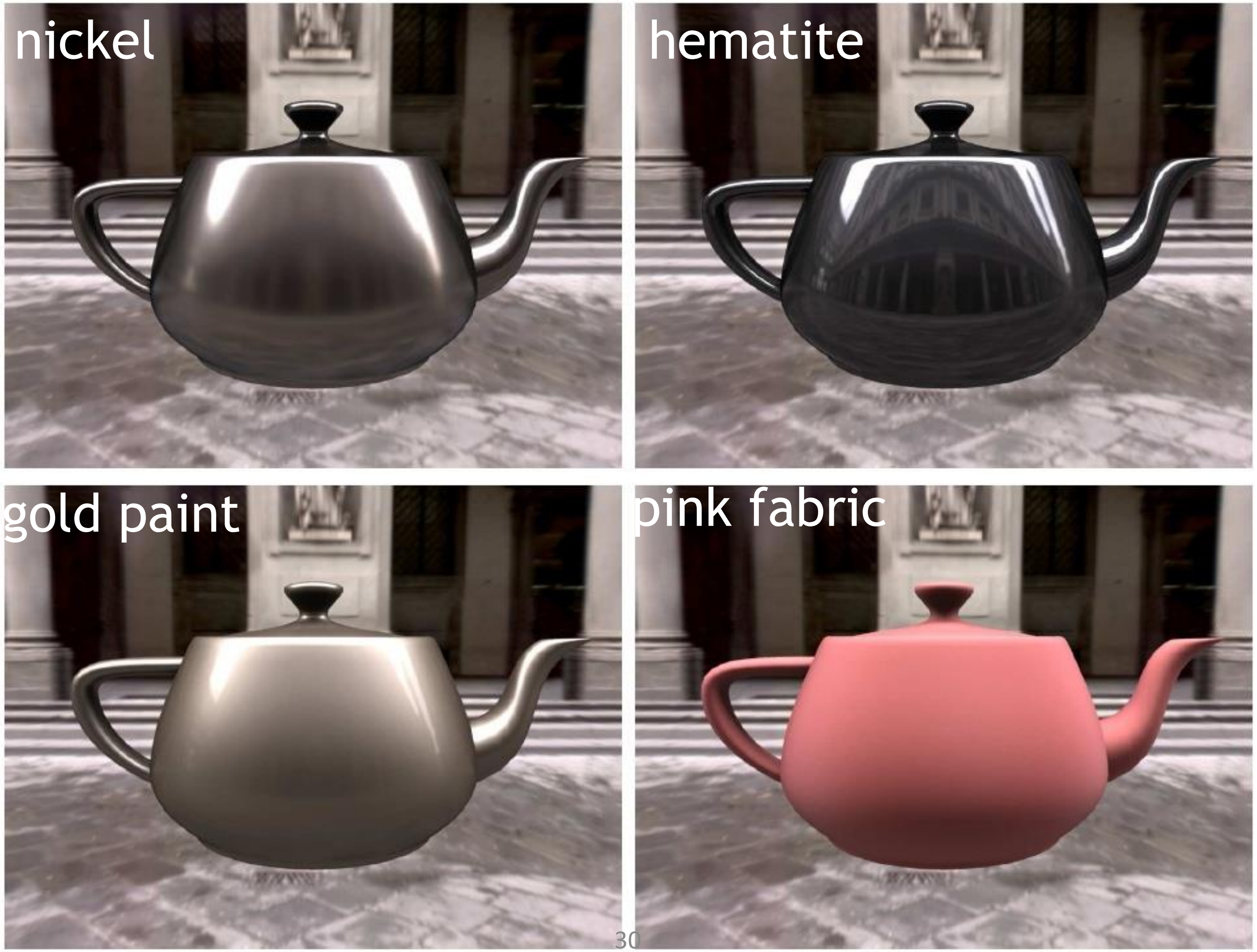
Acquisition



130 materials were scanned;
100 of them shown here

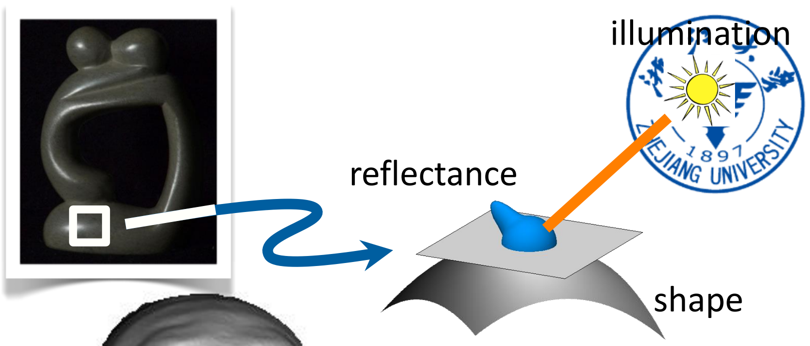
MERL BRDF database, freely available online

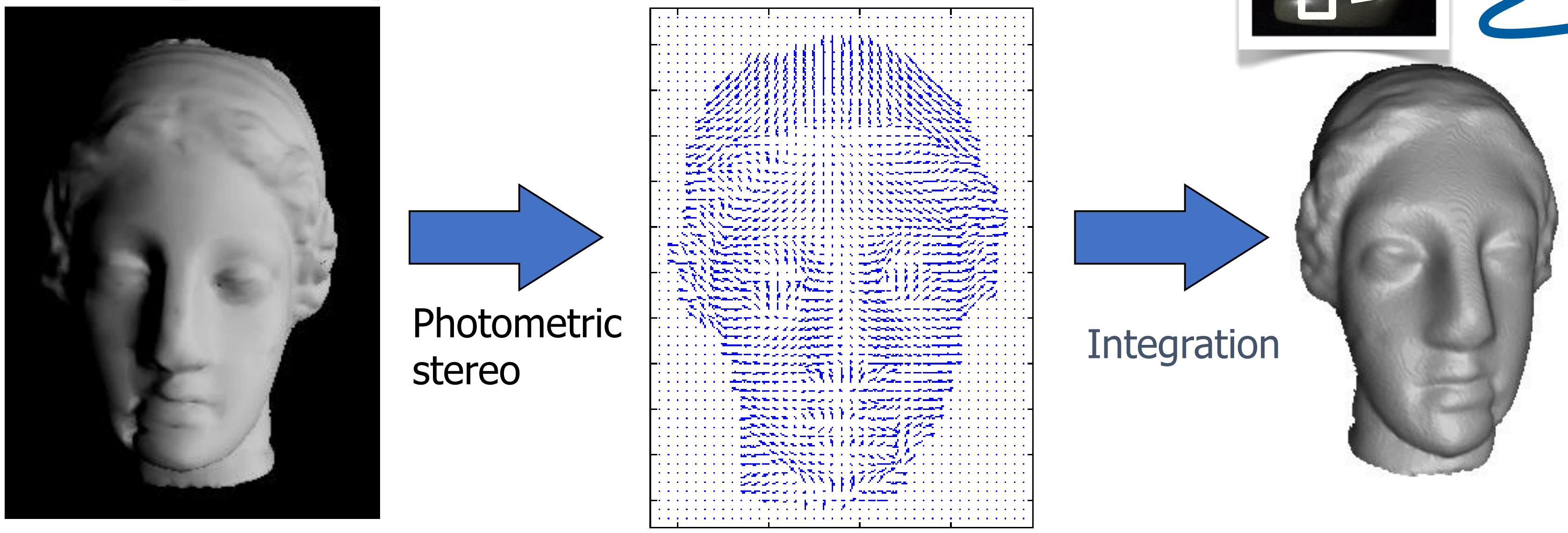
Tabulated BRDF



Questions?

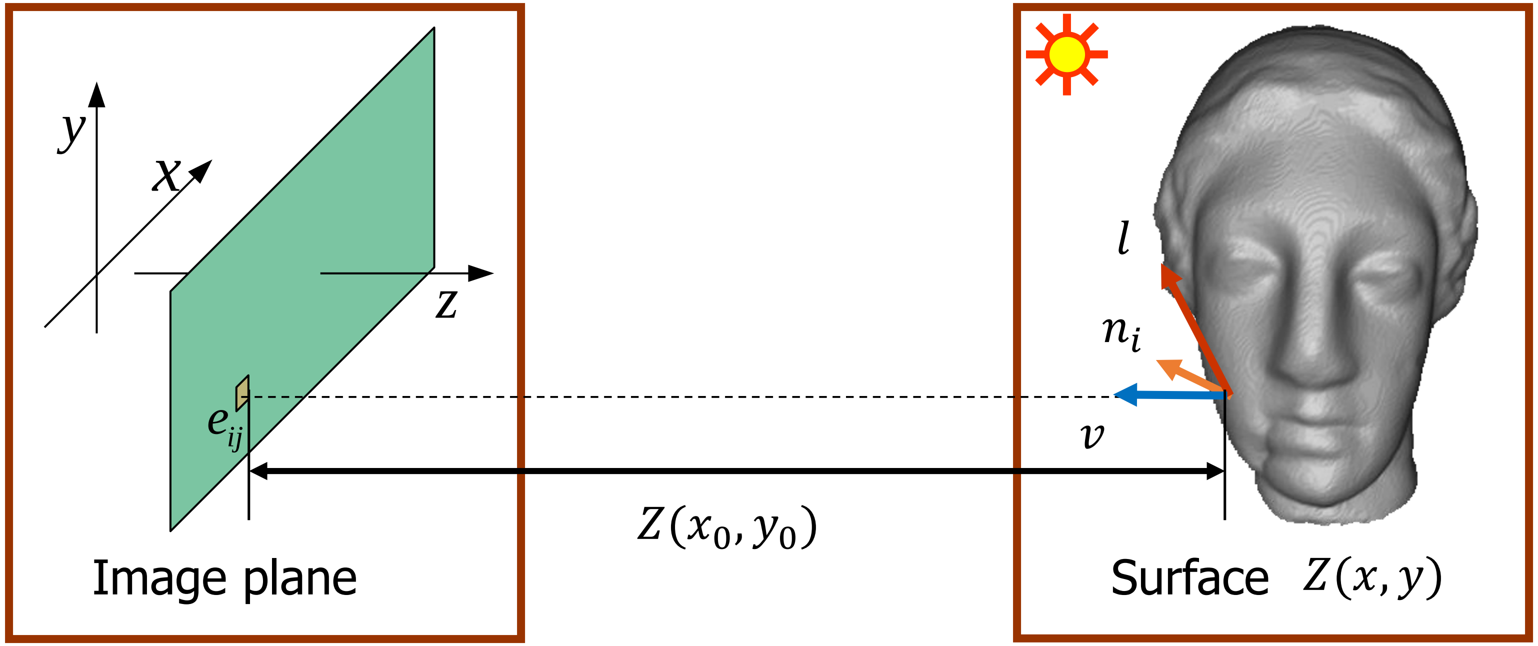
Photometric Stereo (Capturing Shapes)





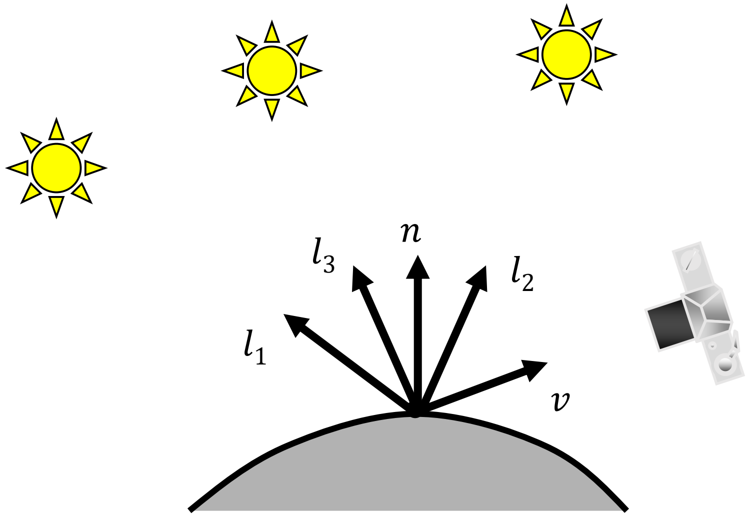
- Radiance (pixel intensity) is determined by shape, lighting, and BRDF
- Capture shape from images with known BRDF and lighting
- Often captured with a fixed camera and a moving light
  - Often assume Lambert’s reflectance (pure diffuse material)
  - Need careful calibration of lighting
    (also a darkroom to avoid inter-reflection, e.g. from the white walls)

Typical Assumptions



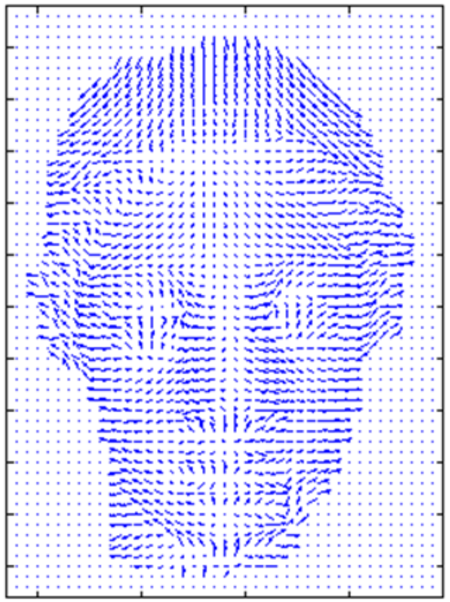
- Lambert's reflectance model
- Camera centered coordinate system
- Orthographic camera (v is the same for all pixels)
- Directional illumination (l is the same for all pixels)

Lambertian Photometric Stereo



$$I_1 = \rho \boldsymbol{n} \cdot \boldsymbol{l_1}$$
$$I_2 = \rho \boldsymbol{n} \cdot \boldsymbol{l_2}$$
$$I_3 = \rho \boldsymbol{n} \cdot \boldsymbol{l_3}$$

By the Lambert's reflectance model



Write in a matrix equation:

$$\underbrace{\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}}_{\substack{\mathbf{I} \\ 3\times 1}} = \underbrace{\begin{pmatrix} \boldsymbol{l}_1^T \\ \boldsymbol{l}_2^T \\ \boldsymbol{l}_3^T \end{pmatrix}}_{\substack{L \\ 3\times 3}} \underbrace{\rho \boldsymbol{n}}_{\substack{\mathbf{b} \\ 3\times 1}} \xrightarrow{L \text{ is known}}$$

$$\mathbf{b} = L^{-1}\mathbf{I}$$
$$\rho = \|\mathbf{b}\|$$
$$n = \frac{1}{\rho}\mathbf{b}$$

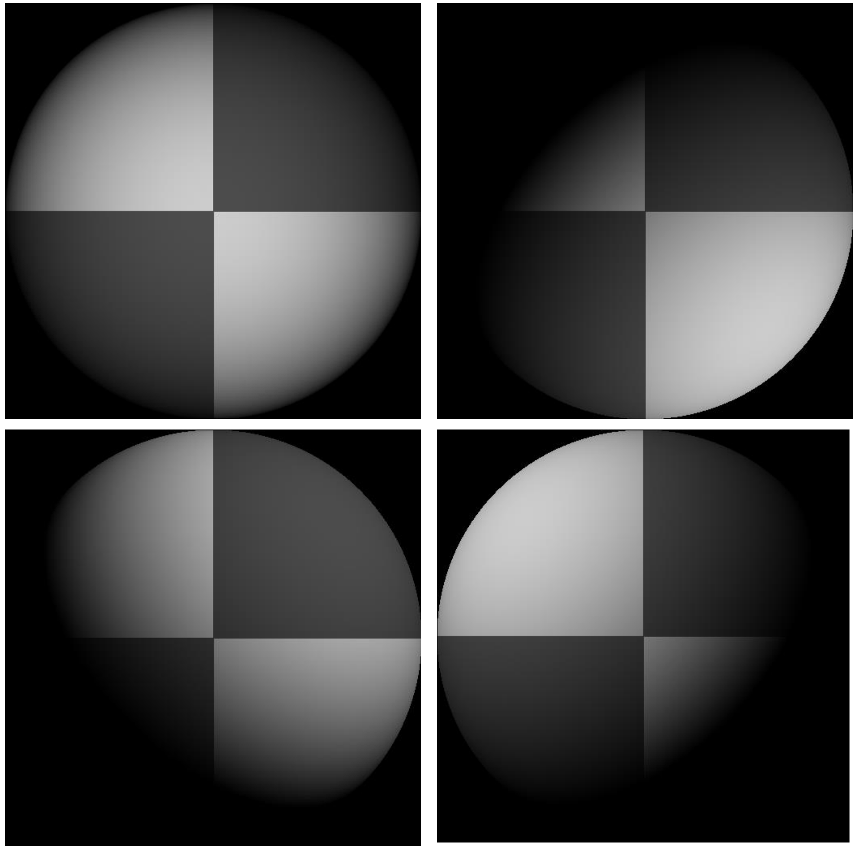
Dealing with Shadows

- The linear formulation, $I = \rho\, \boldsymbol{n} \cdot \boldsymbol{l}$, does not consider shadows
- Pixels in "attached shadows" have zero intensity, $I = 0$, while $\boldsymbol{n} \cdot \boldsymbol{l} < 0$
- A better formulation is nonlinear:

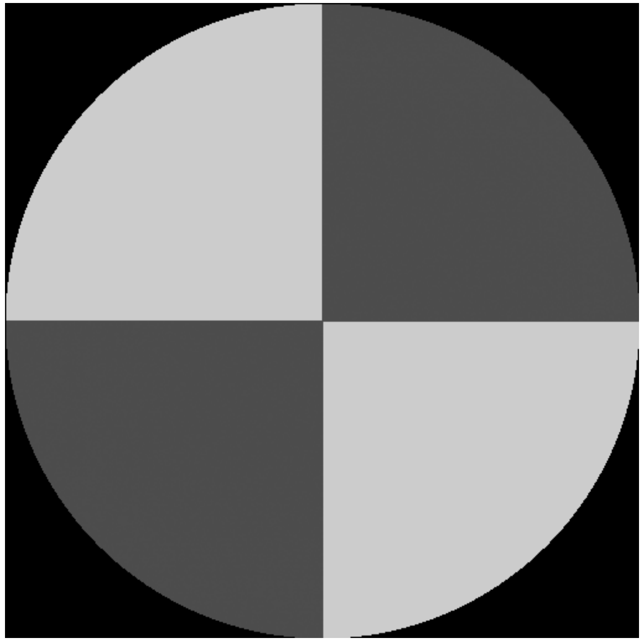
$$I = \max(0, \rho\, \boldsymbol{n} \cdot \boldsymbol{l})$$

  - But $\max(0,\cdot)$ is a nonlinear function, difficult to fit
- A simple way to deal with shadows and keep linear formulation
  - At each pixel, there are multiple observations $I_1, I_2, \cdots I_K$
  - Suppose K is large and the variation in lighting directions is large
    - So there are enough observations in $I_1, I_2, \cdots I_K$ that are free from shadows
  - Thus, we can sort $I_1, I_2, \cdots I_K$ by their values and discard the 20% darkest

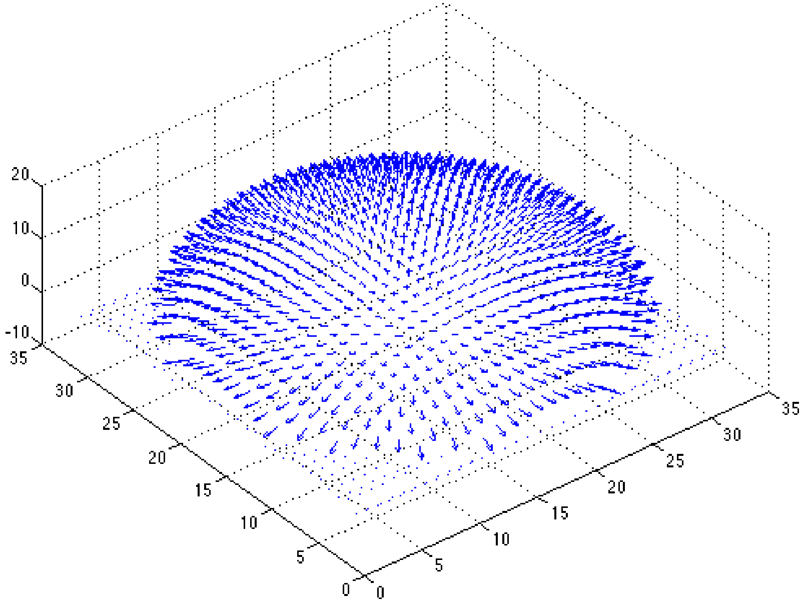
Example Figures



Input images



Albedo ρ



Normal

Questions?

Depth from Normals (Method I)

- Suppose the surface is $(x, y, Z(x, y))$
- The surface normal then should be,

$$\boldsymbol{n}(x,y) = \frac{1}{\sqrt{Z_x^2 + Z_y^2 + 1}} \begin{pmatrix} -Z_x \\ -Z_y \\ 1 \end{pmatrix}$$

- If we denote the normal as,

$$\boldsymbol{n}(x,y) = \begin{pmatrix} n_1(x,y) \\ n_2(x,y) \\ n_3(x,y) \end{pmatrix}$$

- Then we obtain the following partial derivatives:

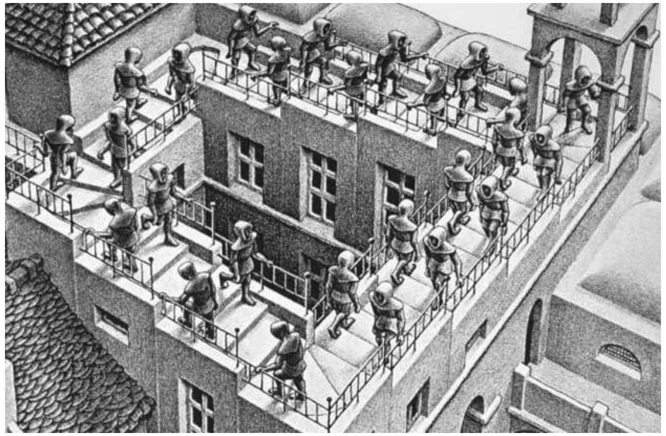
$$Z_x(x,y) = -n_1(x,y)/n_3(x,y)$$

$$Z_y(x,y) = -n_2(x,y)/n_3(x,y)$$

Depth from Normals (Method I)

- We can now recover the surface height at any point by integration along some path, e.g.

$$Z(x,y) = \int_0^x Z_x(s,y)ds + \int_0^y Z_y(x,t)dt + c$$



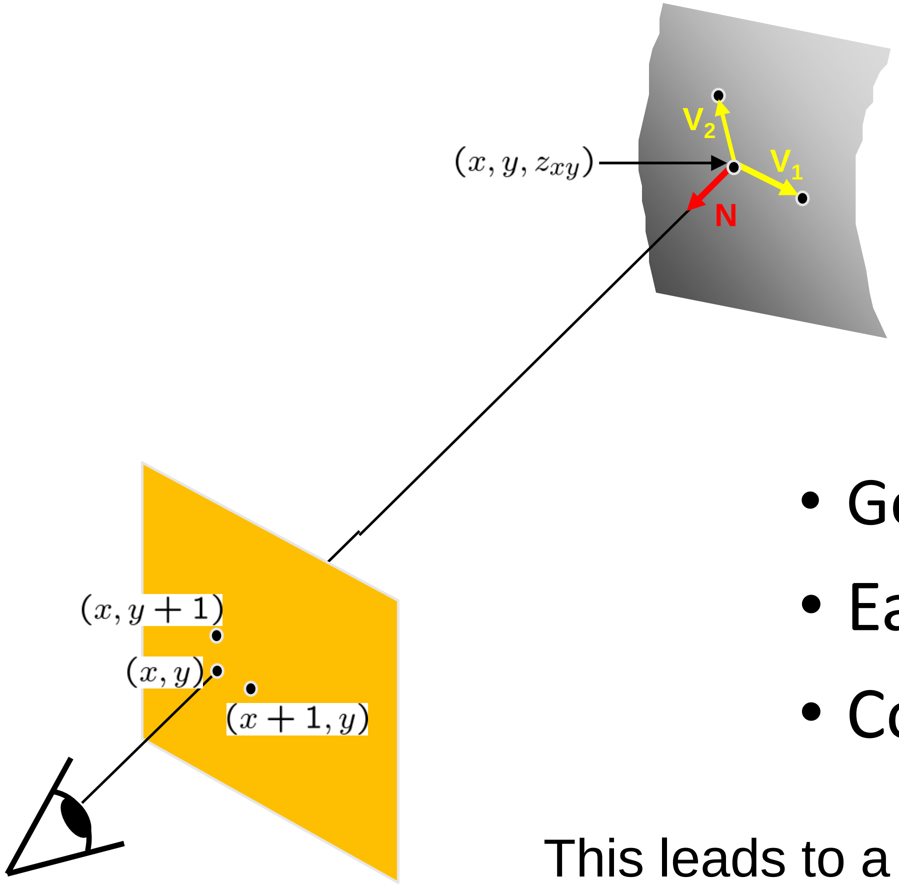
- This method never works on real data. Why?
- The recovered normal is too noisy!
- Recall that mixed second partials are equal --- this gives us a **check.** We must have:

$$\frac{\partial Z_x(x,y)}{\partial y} = \frac{\partial Z_y(x,y)}{\partial x}$$

(or they should be similar, at least)

- Due to imaging and estimation noise, this almost never happens.

Depth from Normals (Method II)



- The tangent vector $\boldsymbol{v}_1$ is perpendicular to $\boldsymbol{n}$

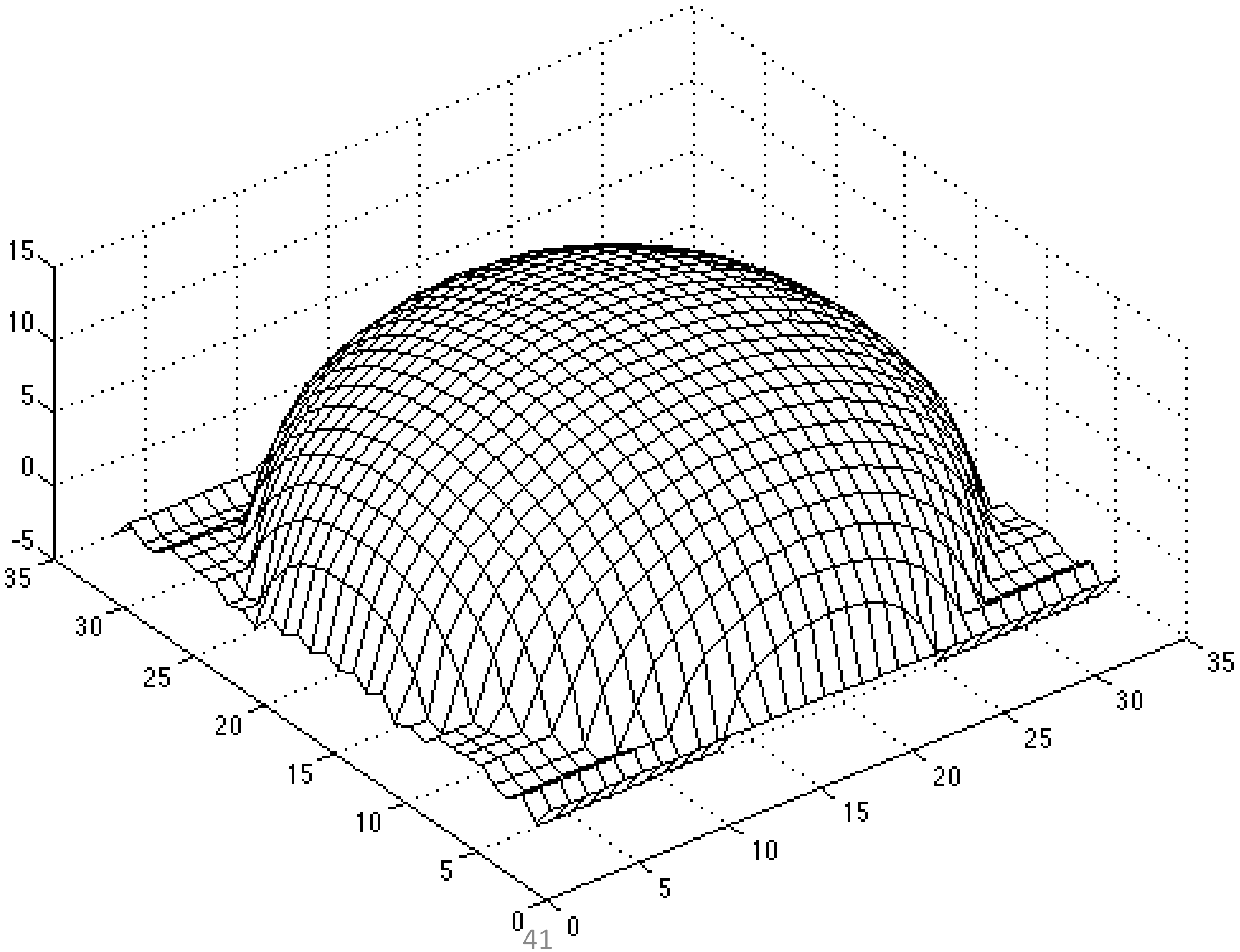
$$\begin{aligned}\boldsymbol{v}_1 &= \big(x+1, y, Z(x+1, y)\big) - (x, y, Z(x, y)) \\ &= (1, 0, Z(x+1, y) - Z(x, y)\ )\end{aligned}$$

$$\begin{aligned}0 &= \boldsymbol{n} \cdot \boldsymbol{v}_1 \\ &= (n_1, n_2, n_3) \cdot (1, 0, Z(x+1, y) - Z(x, y)) \\ &= n_1 + n_3(Z(x+1, y) - Z(x, y))\end{aligned}$$

- Get a similar equation for $\boldsymbol{v}_2$
- Each normal gives two linear constraints on Z
- Compute Z values by solving a matrix equation

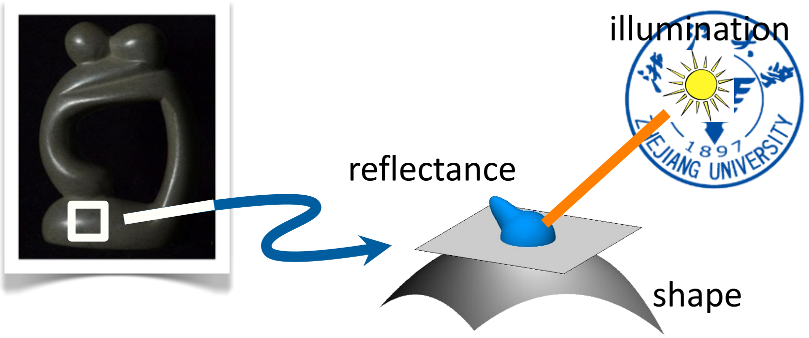
This leads to a large sparse linear equation.
Often solved by the Conjugated Gradient algorithm.

Surface Recovered



Questions?

Capturing Lighting

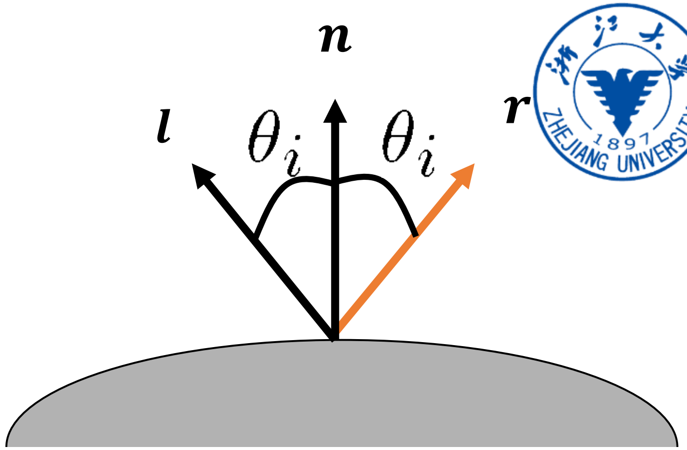


- Radiance (pixel intensity) is determined by shape, lighting, and BRDF
- Capture lighting from images with known BRDF and shape
- Often captured with mirror spheres
    - Known shape (sphere) and known BRDF (mirror)

Capture a Directional Light

- For example, to get the matrix L in photometric stereo (page 34)
- Capture a shiny sphere in the scene
  - the location of the highlight tells the lighting direction

Capture a Directional Light

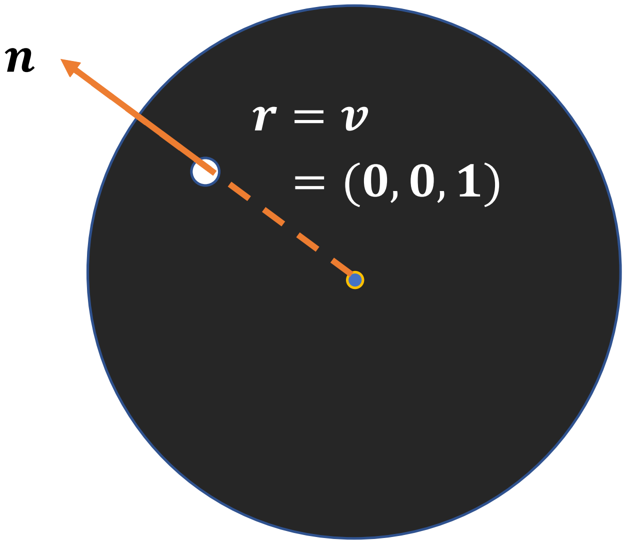


$$\boldsymbol{l} = 2(\boldsymbol{n} \cdot \boldsymbol{r})\boldsymbol{n} - \boldsymbol{r}$$

- For a mirror sphere, light is reflected about $\boldsymbol{n}$

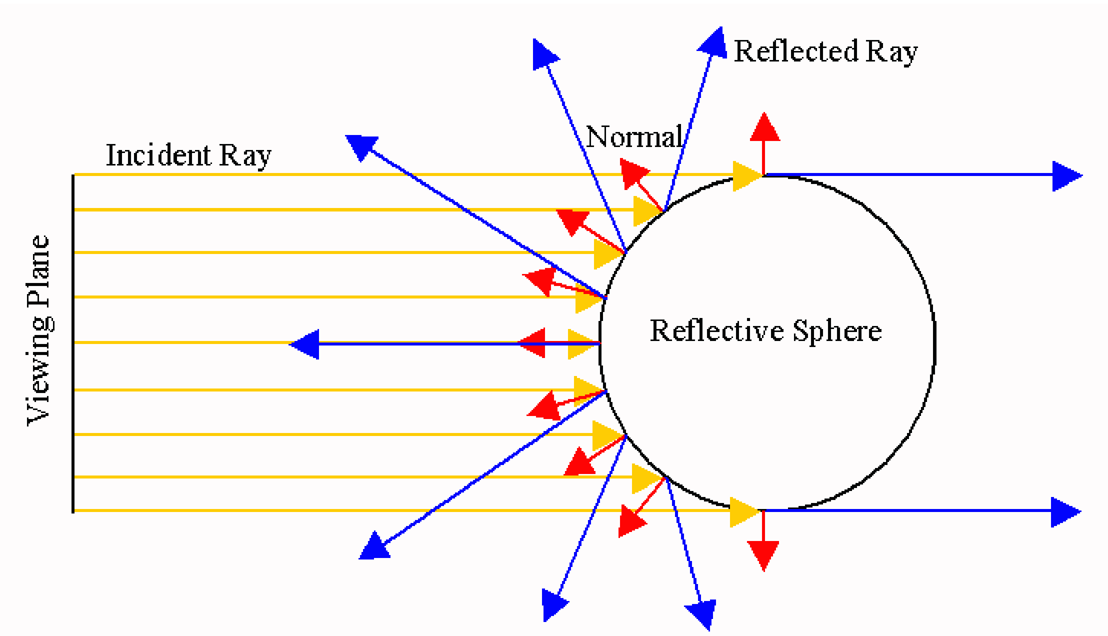
$$L_o = \begin{cases} L_i & \text{if } \boldsymbol{v} = \boldsymbol{r} \\ 0 & \text{otherwise} \end{cases}$$

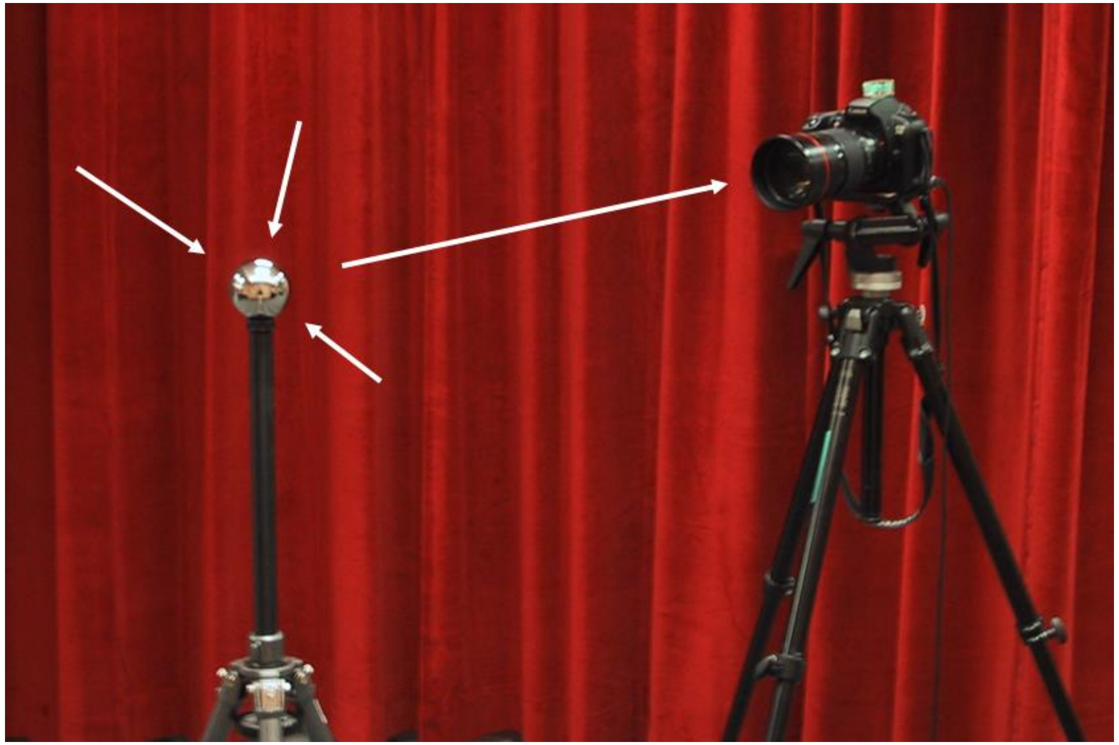
- The light source is seen at a pixel where $\boldsymbol{v} = \boldsymbol{r}$
- Assume orthographic camera
  - $\boldsymbol{v} = (0,0,1)$ for all pixels
- So if we further know $\boldsymbol{n}$, we can compute $\boldsymbol{l}$
  - normal of each point on a sphere can be determined
  - (by some simple geometry, try to derive it yourself)



Capture an Environment Light

- Trace back all light rays reflected from a mirror sphere
- Still assume orthographic camera

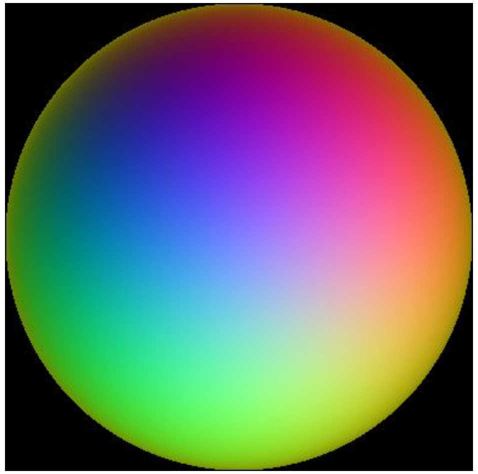




Mirror ball -> equirectangular

Mirror ball

Normals



Reflection vectors

Equirectangular (longitude & latitude)

Applications of Captured Lighting

- Rendering virtual objects into real scenes (e.g. in AR)

from Terminator 2

HDR Environment Lighting

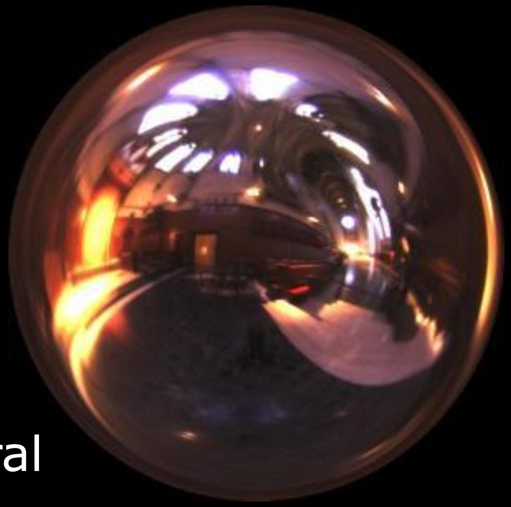
- HDR is needed so that light probes capture full range of radiance

Real-World HDR Lighting Environments

Funston
Beach

Eucalyptus
Grove

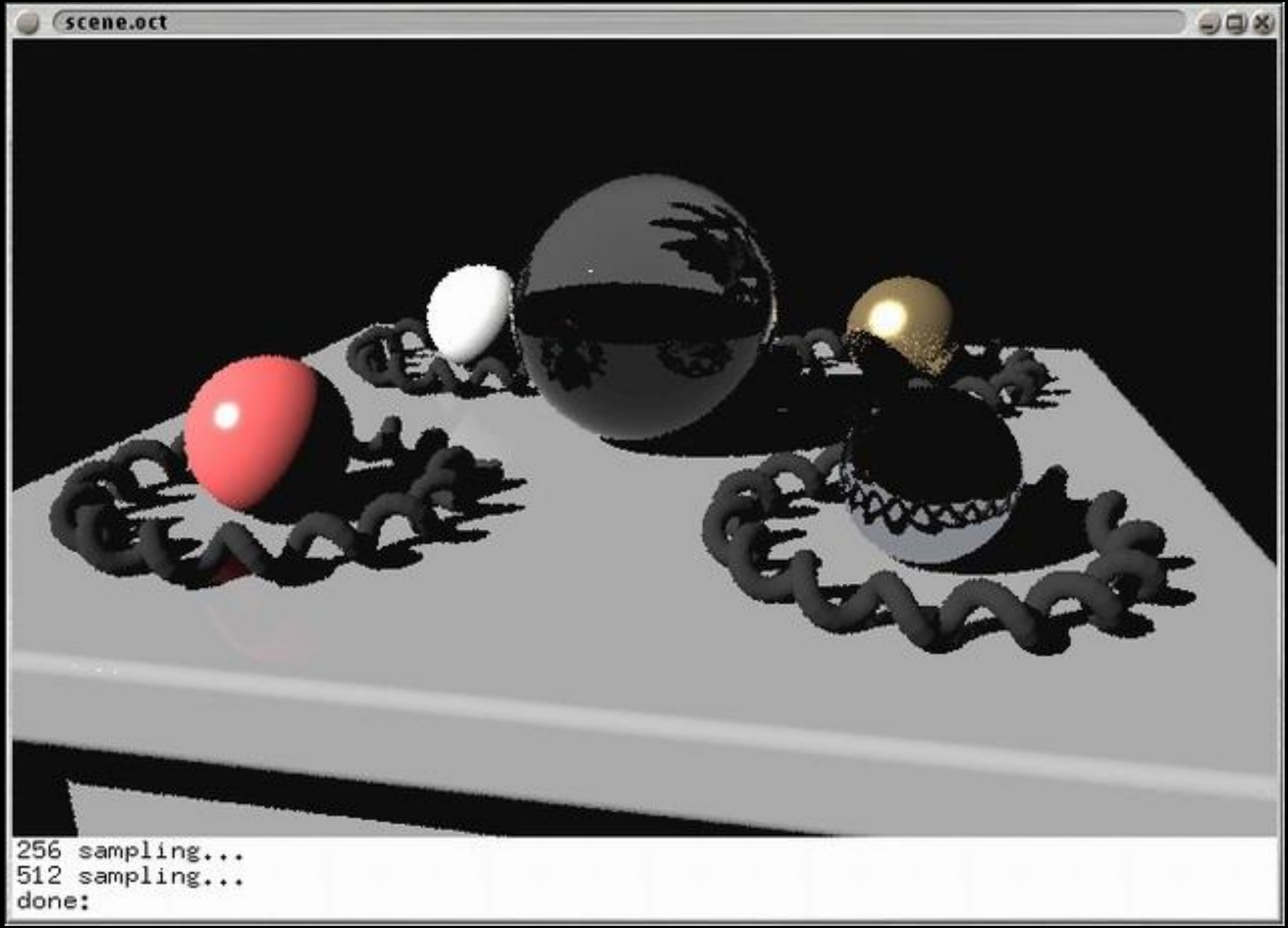
Uffizi
Gallery



Grace
Cathedral

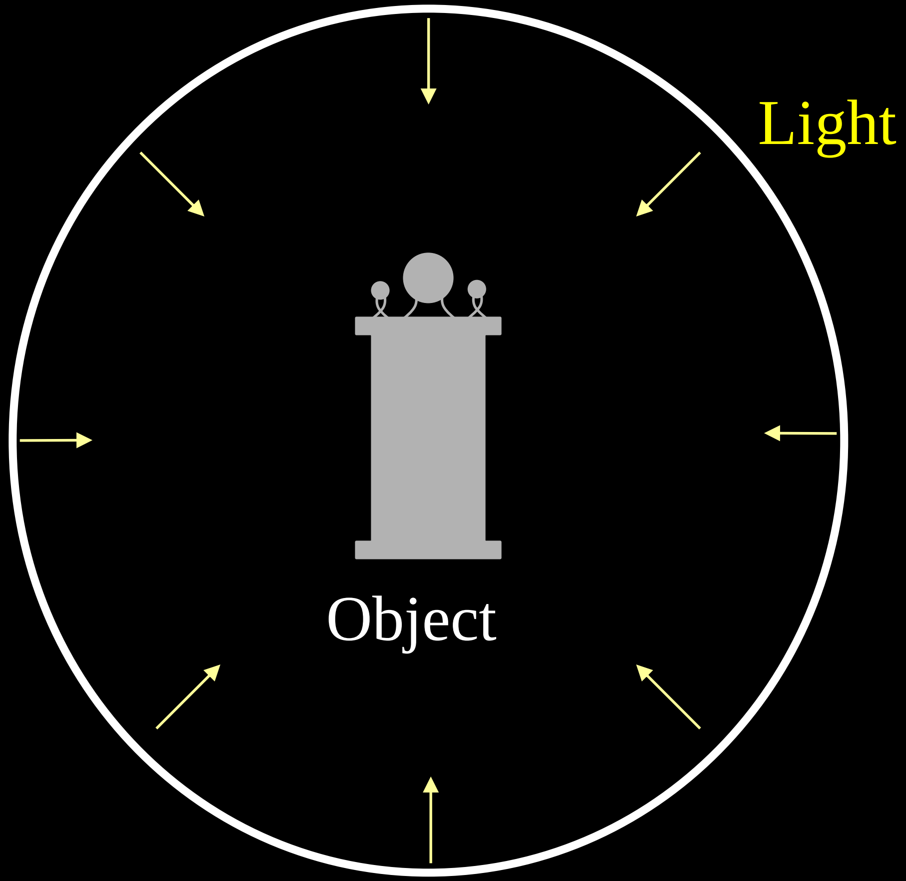
Lighting Environments from the Light Probe Image Gallery:
http://www.debevec.org/Probes/

Slides from Paul Debevec



CG Objects Illuminated by a Traditional CG Light Source

Illuminating Objects using Measurements of Real Light



Environment assigned "glow" material property in Greg Ward's RADIANCE system.

http://radsite.lbl.gov/radiance/

Paul Debevec. A Tutorial on Image-Based Lighting. IEEE Computer Graphics and Applications, Jan/Feb 2002.

http://www.youtube.com/watch?v=EHBgkeXH9lU

Questions?