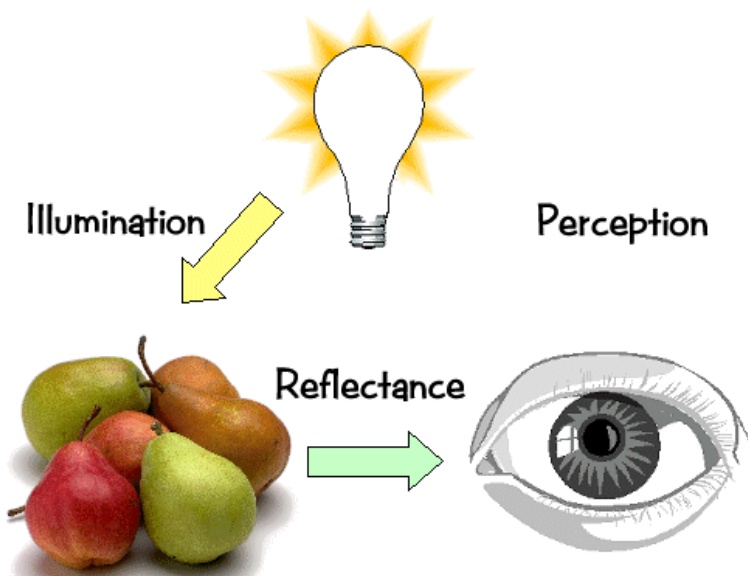


3. Reflectance & Lighting



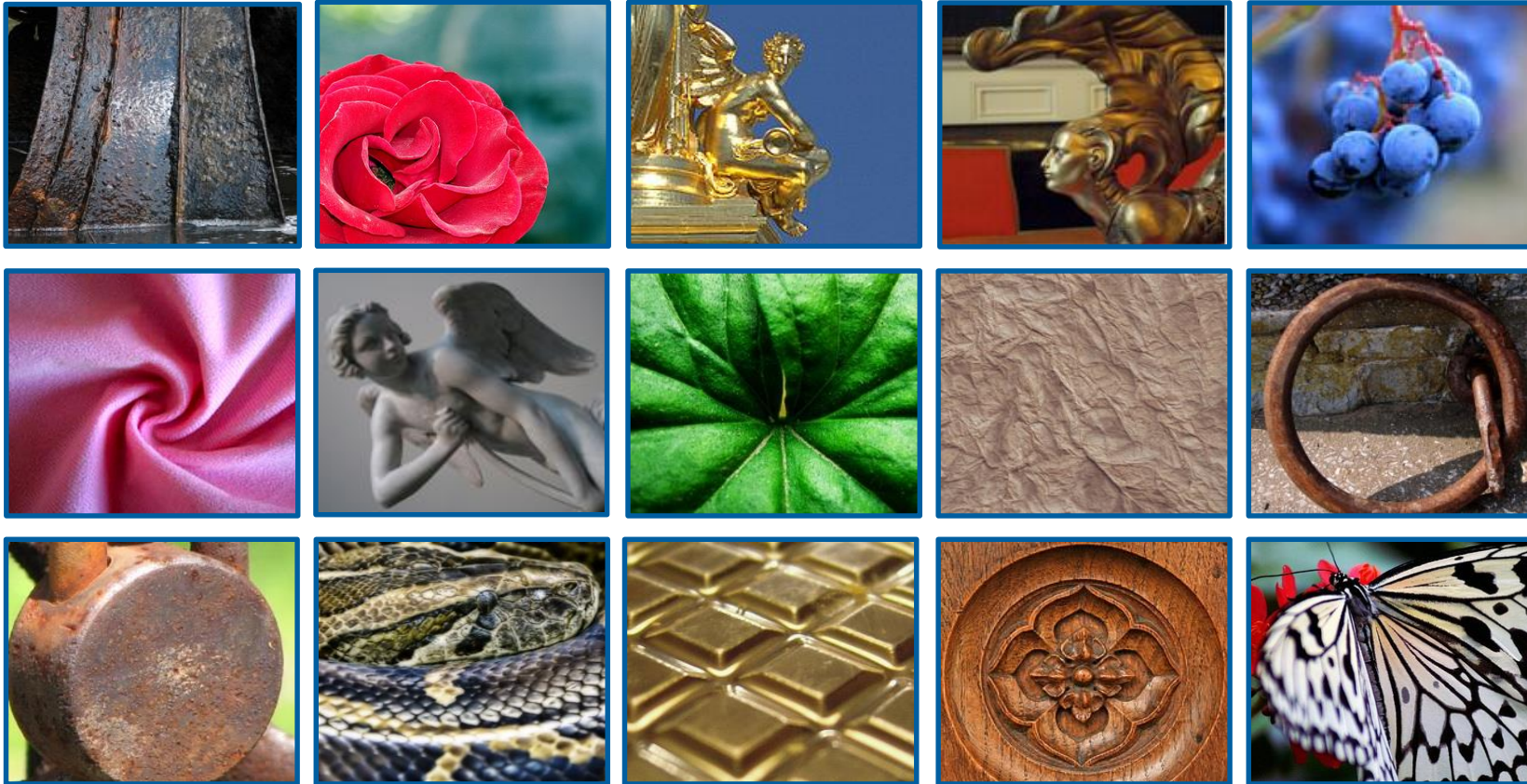
Lights, Material/Reflectance, and Geometry

- The radiance at a pixel is determined by:
 - Lighting (direction & intensity)
 - Material/reflectance
 - Local shape (mainly surface normal direction)



Light at Surfaces

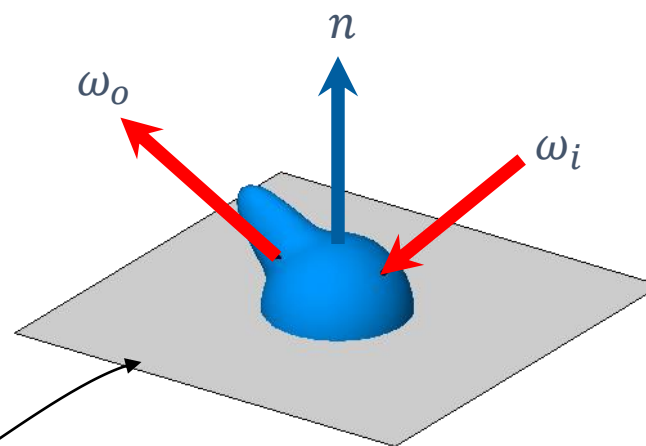
- Many effects when light strikes a surface -- could be:
 - Reflected, refracted, scattered, absorbed, etc.
 - We focus on reflectance today for simplicity



Material/Reflectance

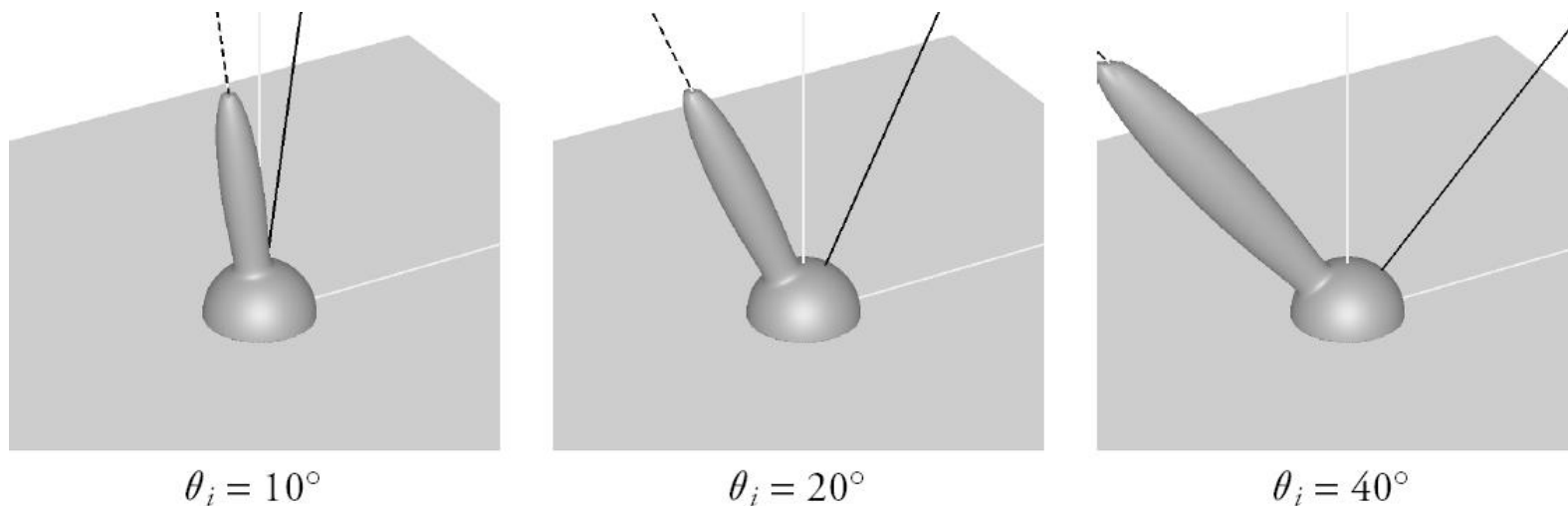
- *Reflectance* is all about the way light interacts with surfaces
- It is an entire field of study on its own
- The most important quantity is the BRDF (Bi-directional Reflectance Distribution Function)
- A BRDF $\rho(\omega_o, \omega_i)$ is a function of two directions
 - Incident lighting direction ω_i
 - Reflected lighting direction ω_o

Both directions are defined in a local coordinate system where typically the surface normal direction n is the z -axis



The BRDF

- It describes how reflected lights are distributed
- It is a PDF function for each fixed ω_i (the distribution of reflected energy)
- This distribution changes when the incoming ray changes



visualize a BRDF as a function of ω_o for a fixed ω_i ;
the radius along each direction is set to the radiance of the
reflected light at that direction.



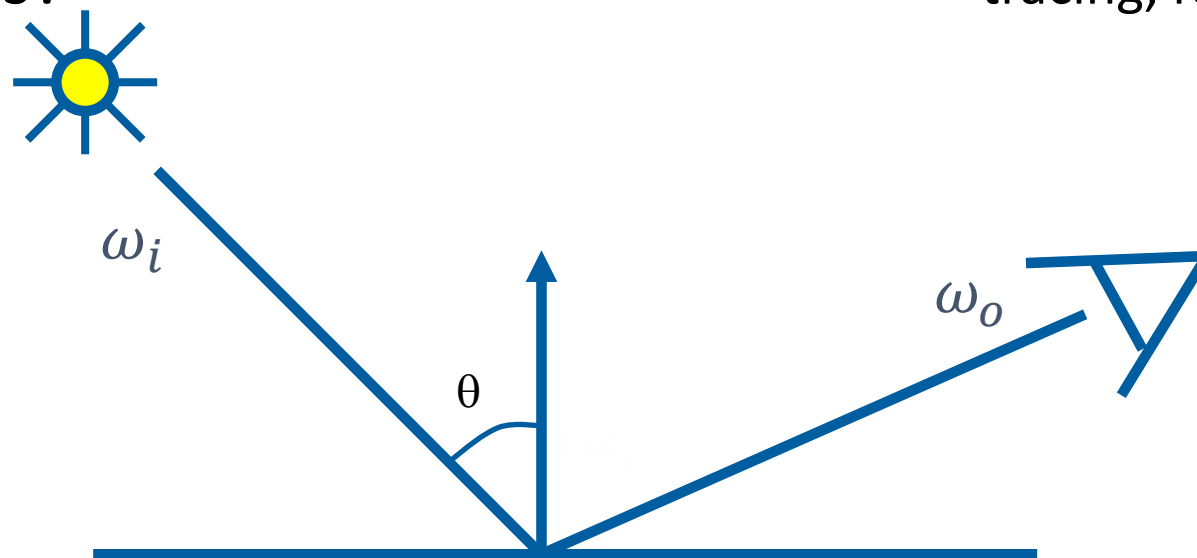
Local Assumption

- BRDFs assume reflectance is local: all light leaving a point depends ONLY on the light arriving at that point
- It ignores many non-local behavior, e.g.
 - Translucency: semi-transparent materials, e.g. marble, human skin, etc
 - Fluoresce: absorbing lighting in one wavelength and emit in a different wavelength
- In this class, we further ignore non-local effects, e.g.
 - Inter-reflection
 - Cast shadow

The Rendering Equation

The basis for computer graphics rendering (e.g. ray tracing, radiosity, etc)

- Why study BRDFs?



$$L_o(\omega_o) = \rho_{bd}(\omega_o, \omega_i) L_i(\omega_i) \cos \theta_i$$

Reflected Radiance
(Pixel Intensity)

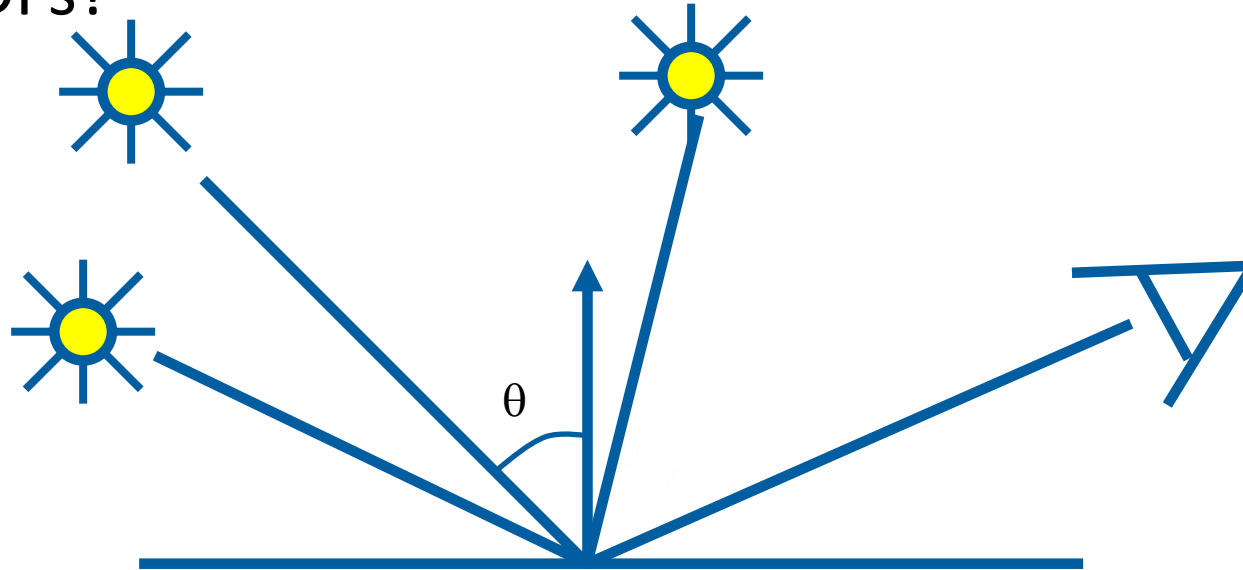
BRDF

Incident
radiance (from
light source)

Cosine of
Incident angle

The Rendering Equation

- Why study BRDFs?



Sum over all light sources

$$L_o(\omega_o) = \sum_i \rho_{bd}(\omega_o, \omega_i) L_i(\omega_i) \cos \theta_i$$

Reflected Radiance
(Pixel Intensity)

BRDF

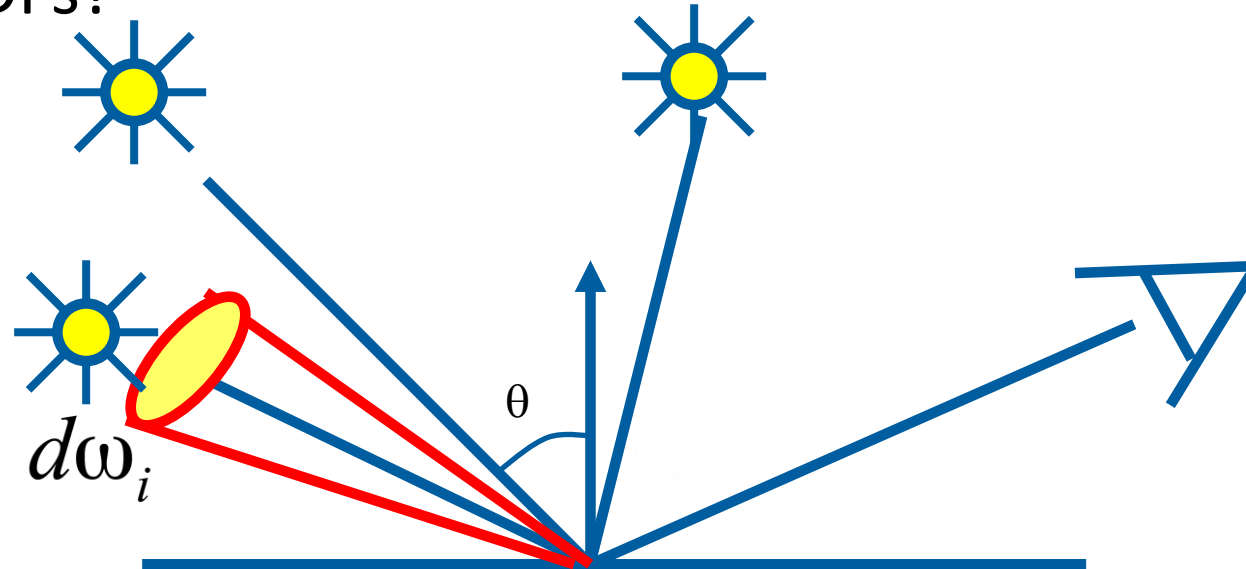
Incident
radiance (from
light source)

Cosine of
Incident angle

The Rendering Equation

The rendering equation consider all points in a scene. What we see here is only one point.

- Why study BRDFs?



Replace sum with integral

$$L_o(\omega_o) = \int_{\Omega} \rho_{bd}(\omega_o, \omega_i) L_i(\omega_i) \cos \theta_i d\omega_i$$

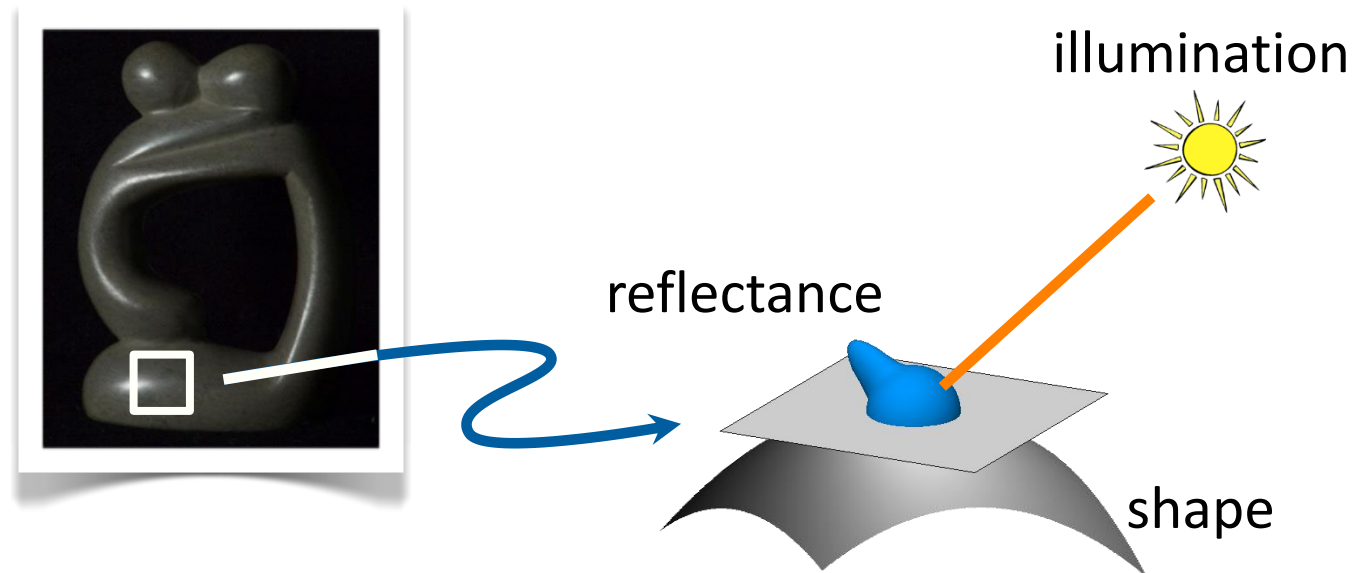
Reflected Radiance
(Pixel Intensity)

BRDF

Incident
radiance (from
light source)

Cosine of
Incident angle

Radiometric Image Analysis

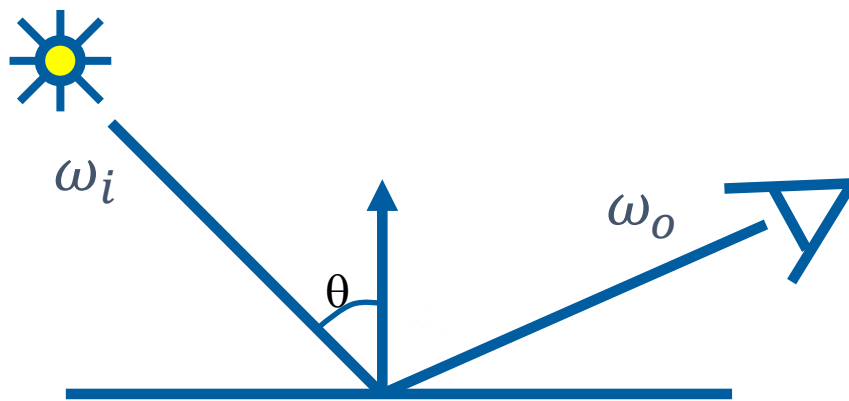


- The rendering equation:
determine irradiance (pixel values) from shape, lighting, and reflectance
- Radiometric image analysis:
recover shape, lighting, or reflectance from irradiance (pixel values)

Radiometric Image Analysis

Typical simplification assumptions:

- Single point light source (simplify the light source)
- No inter-reflection, no cast-shadow (ignore global shape effects)
- Simplified BRDF models (simplify the reflectance)



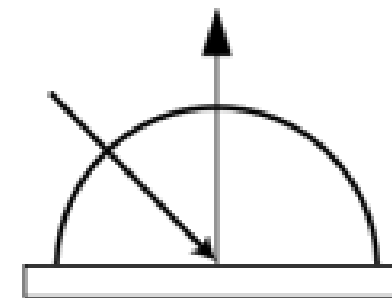
$$L_o(\omega_o) = \rho_{bd}(\omega_o, \omega_i)L_i(\omega_i)\cos \theta_i$$

Questions?

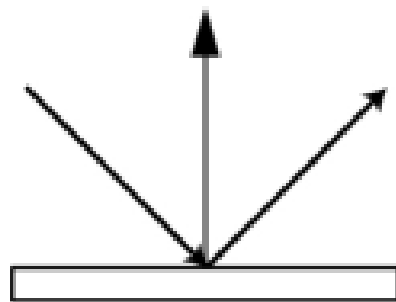


Diffuse & Specular Reflection

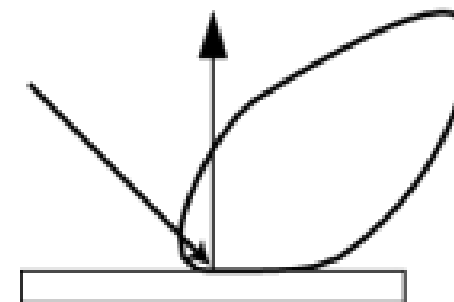
- Diffuse reflection:
 - The surface look the same from all directions (many vision algorithms depend on this!)
 - Matte surfaces
- Specular reflection:
 - The surface look different from different directions (causes troubles to many vision algorithms)
 - Shiny surfaces



ideal diffuse reflection
(e.g. walls)



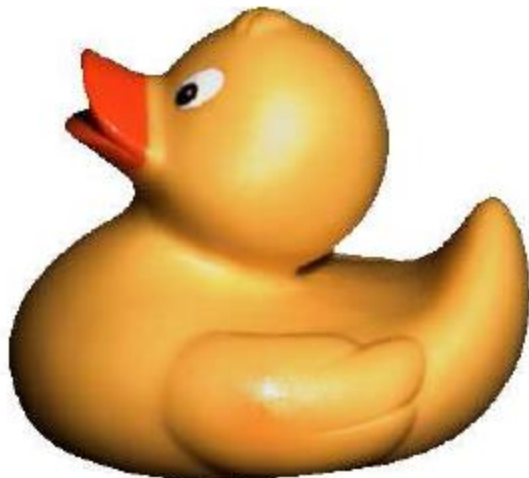
ideal specular reflection (e.g.
mirror)



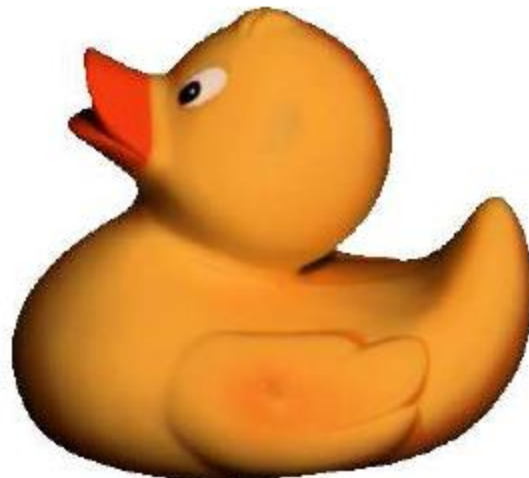
specular reflection (e.g.
plastic, metal, porcelain)

Diffuse & Specular Reflection

- Diffuse reflection:
 - has the same color as the object surface
 - is unpolarized
- Specular reflection:
 - has the same color as the light source
 - has the same polarization as the light source



the original image



diffuse reflection



specular reflection

Lambert's Model (Diffuse Reflection)



Johann H. Lambert

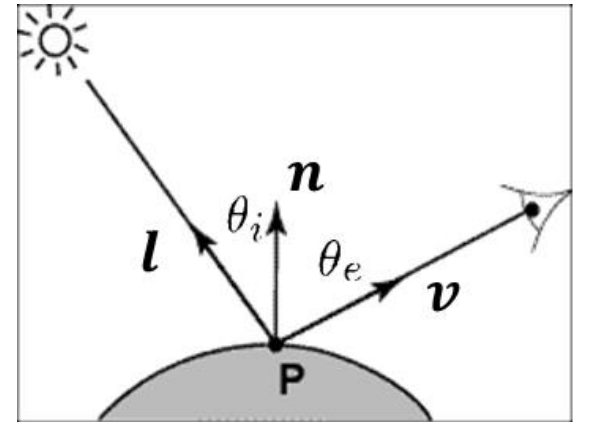
- Empirical mathematic model for diffuse reflection
 - Assume the BRDF is a constant $\rho(\omega_o, \omega_i) = \rho_0$
 - Observed Pixel intensity should be

$$L_o = L_i \rho_0 \cos \theta_i = L_i \rho_0 \mathbf{n} \cdot \mathbf{l}$$

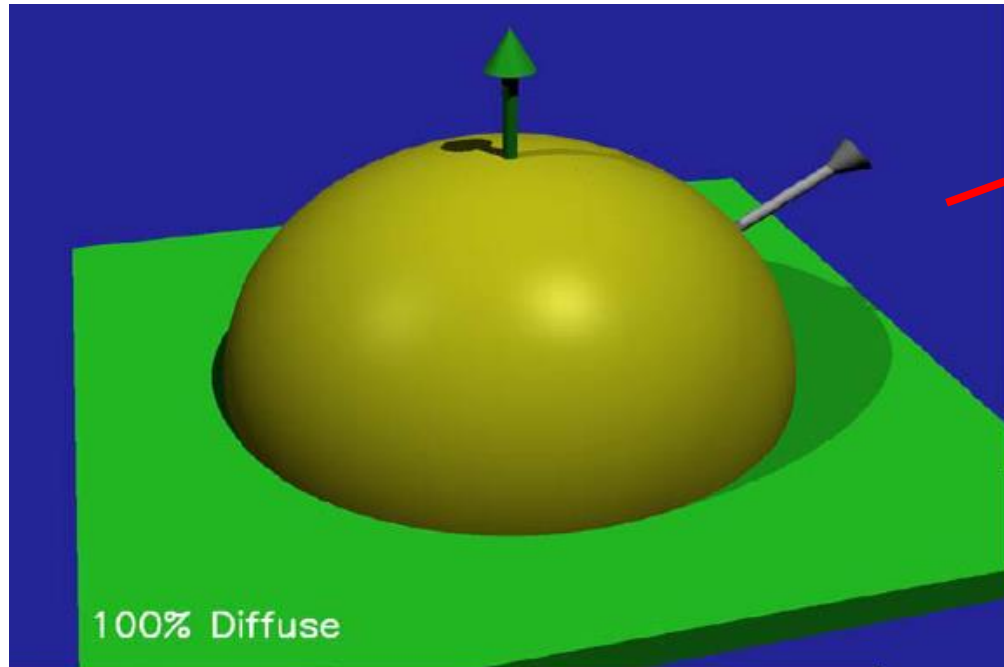
L_i and L_o are intensity of incoming and outgoing light

- Features of this model:

- A pixel's brightness does not depend on viewing direction
- Brightness DOES depend on direction of illumination
- This is the model most commonly used in computer vision (multi-view photo-consistency: the same 3D point look the same across views)



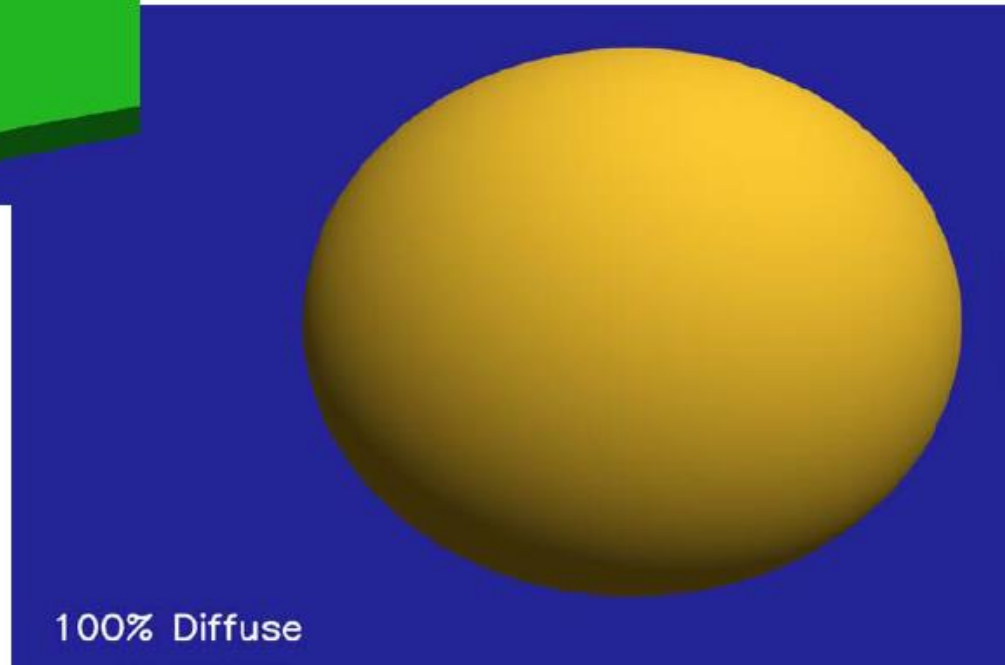
Lambert's Model



plot a BRDF as a function of ω_o for a fixed ω_i

Appearance of a diffuse (dull) sphere

3D plot of reflected intensity

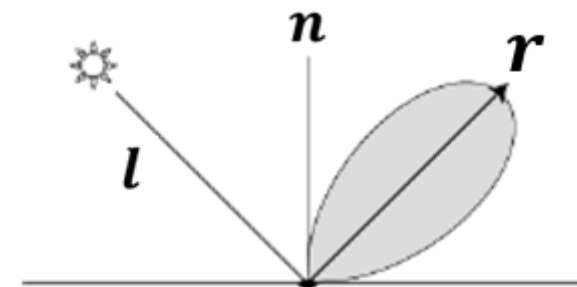


Phong Model (Specular Reflection)

- Mathematic model for specular reflection
 - Assume light is concentrated on the “mirrored direction” \mathbf{r} , $\mathbf{r} = 2(\mathbf{n} \cdot \mathbf{l}) \mathbf{n} - \mathbf{l}$
 - Intensity of light falls off by cosine law
 - Observed Pixel intensity should be

$$L_o = L_i (\mathbf{v} \cdot \mathbf{r})^n$$

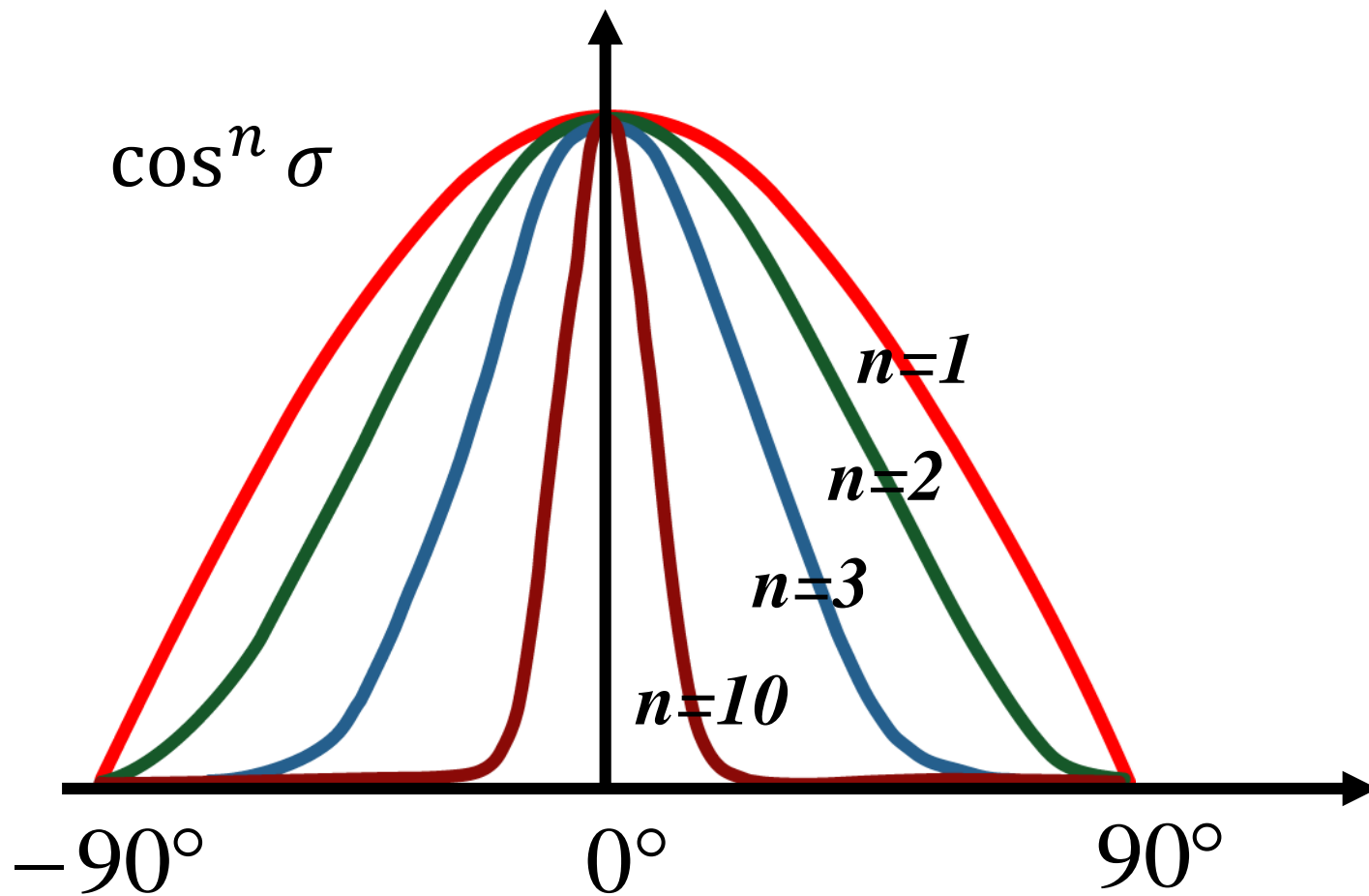
- Features of this model:
 - A pixel’s brightness depends on viewing direction
 - This is an empirical model, not physically correct! (e.g. violate energy conservation)





Phong Model

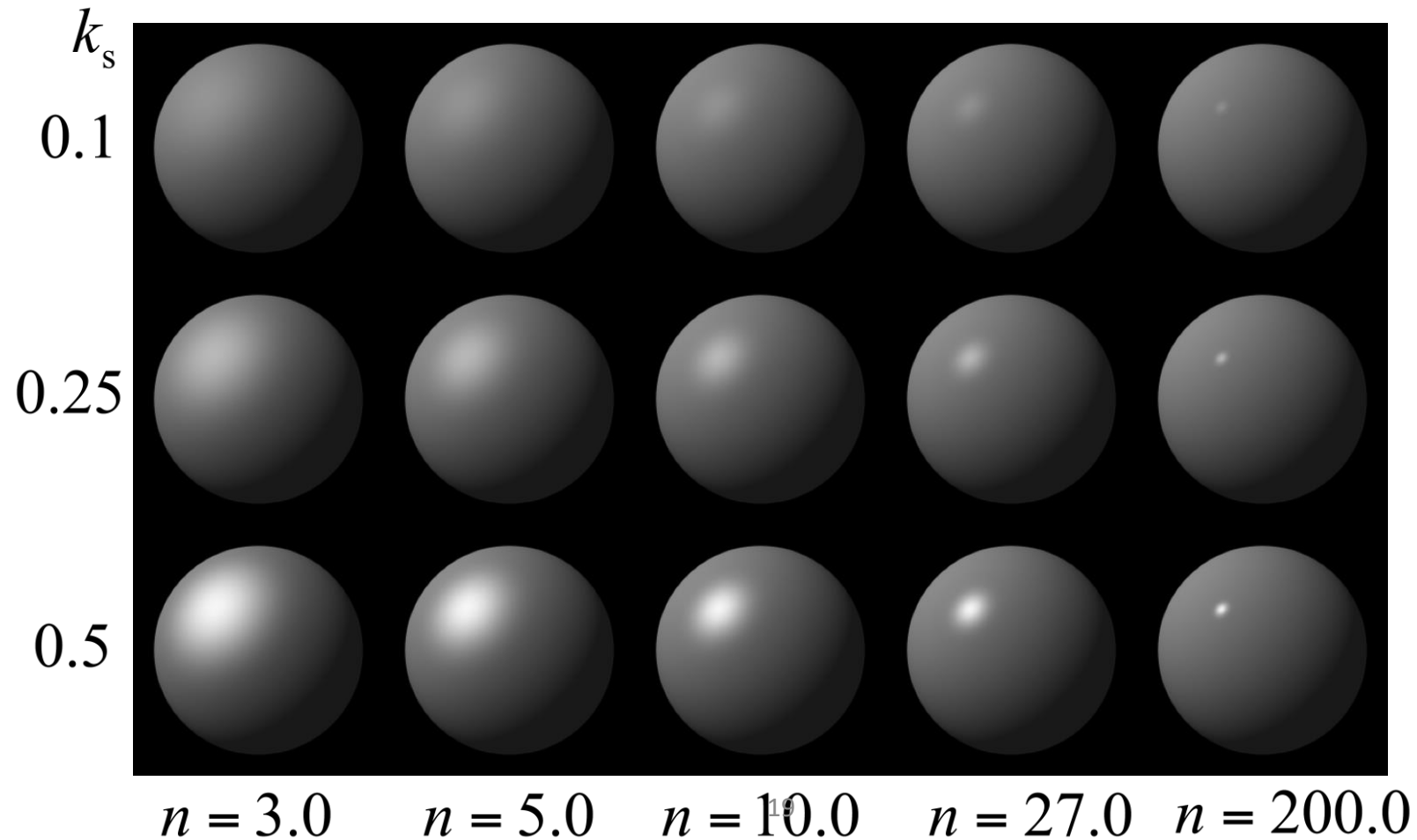
Shininess n controls the size of the highlight spot



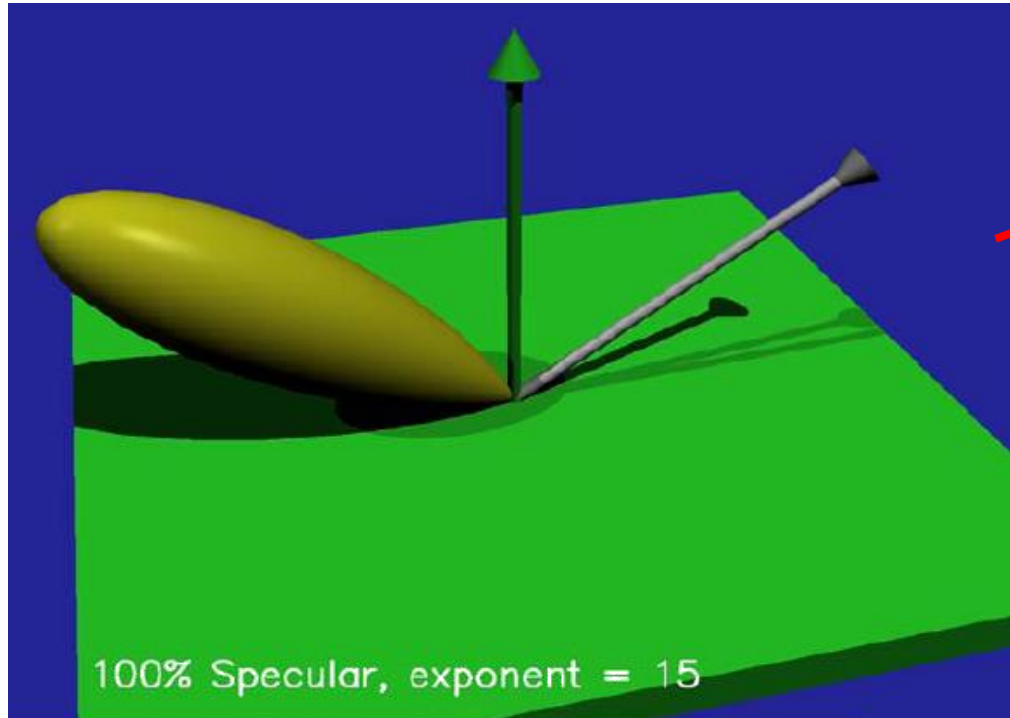
Phong Model

- Linear combination of Lambert's model and Phong Model

$$L_o = k_d \mathbf{n} \cdot \mathbf{l} + k_s (\mathbf{v} \cdot \mathbf{r})^n$$



Phong Model

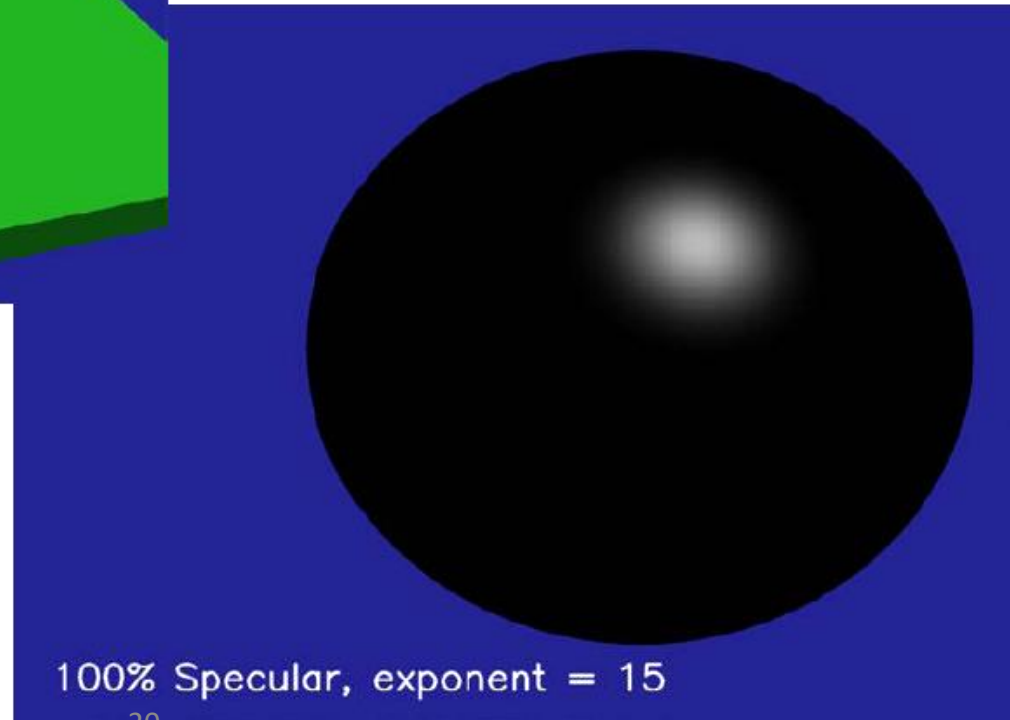


3D plot of reflected intensity

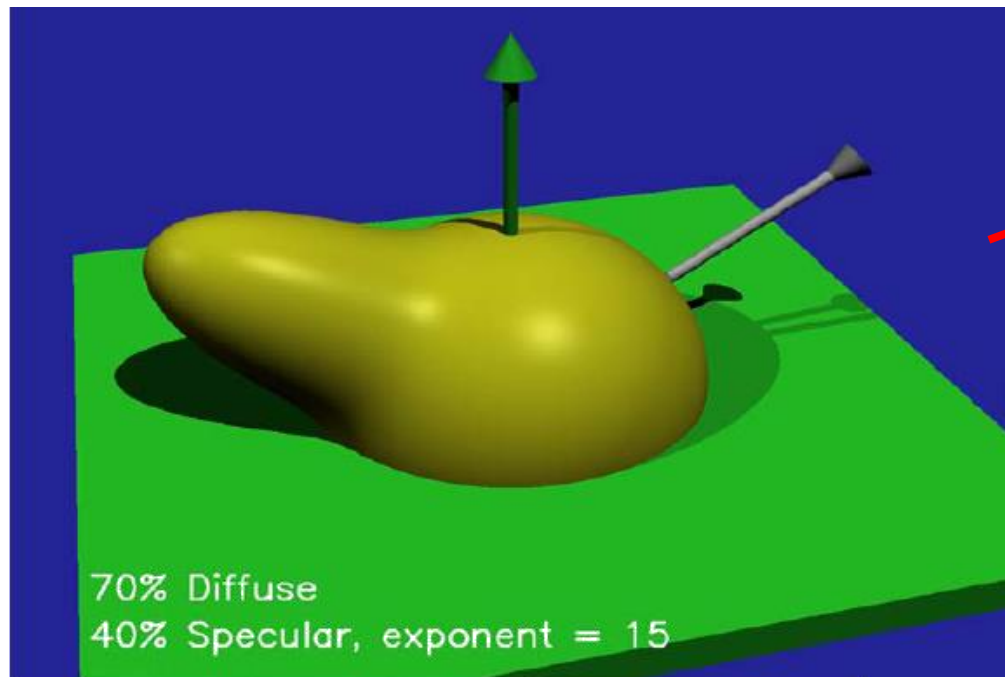
$$n = 15$$

plot a BRDF as a function of ω_o for a fixed ω_i

Appearance of a specular sphere

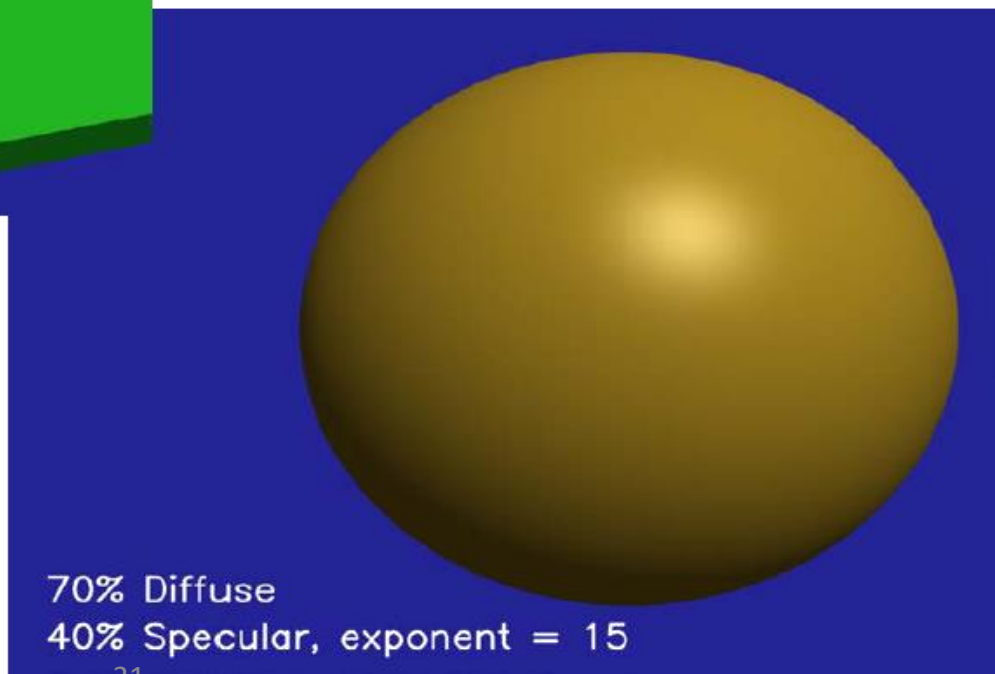


Phong Model



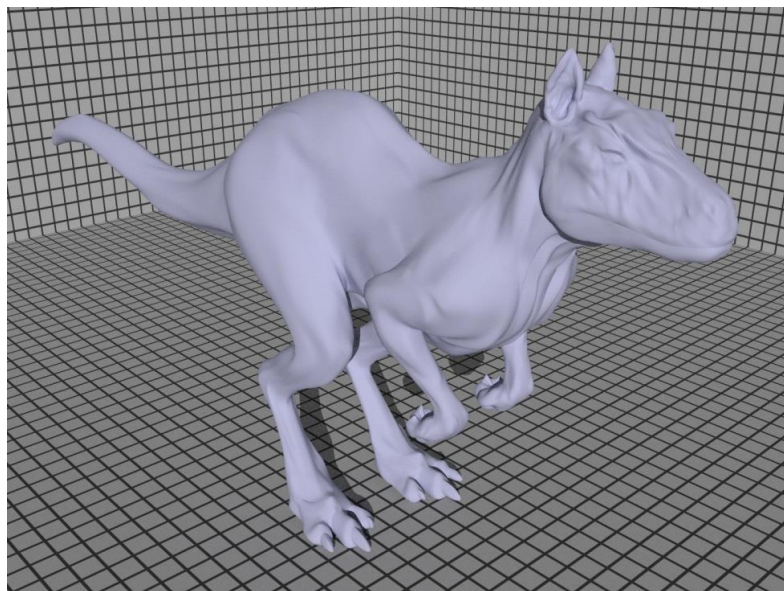
plot a BRDF as a function of ω_o for a fixed ω_i

$$k_d = 0.7$$
$$k_s = 0.4$$
$$n = 15$$

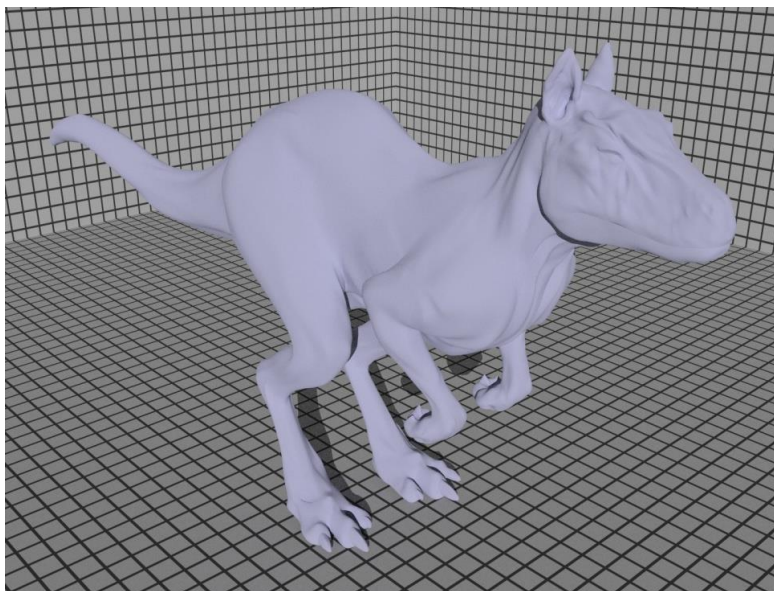


Many More Advanced Models

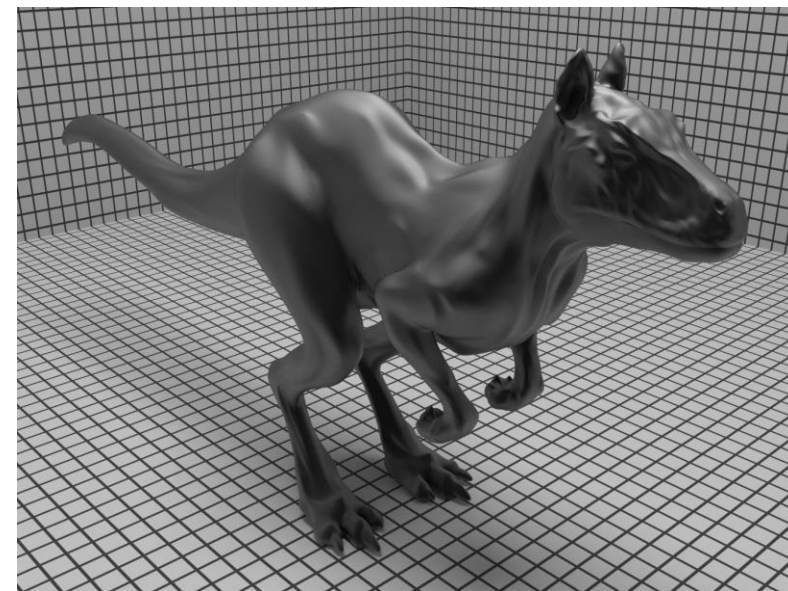
- To describe reflectance more faithfully
 - Oren-Nayar model (a diffuse reflectance model)
 - Cook-Torrance model (a specular reflectance model)
 - Ward's model (a specular reflectance model)



Lambert's model



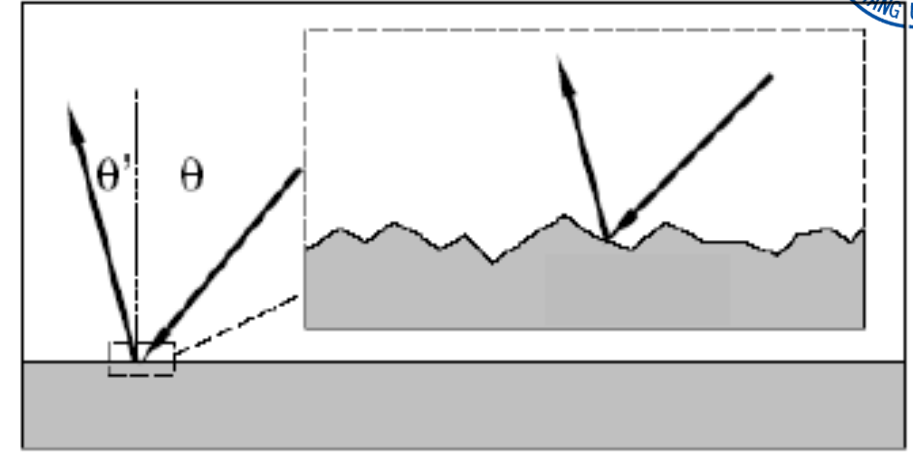
Oren-Nayar model



Cook-Torrance model

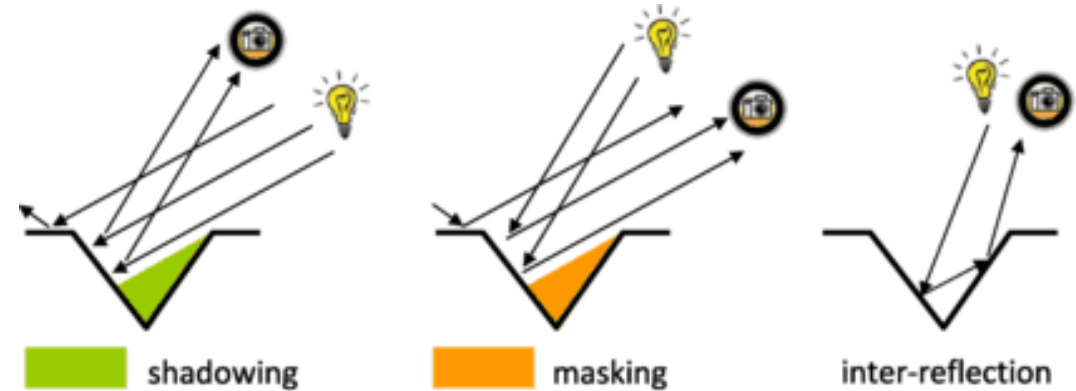
Microfacet Theory

- Assumptions:
 - The surface consists of microfacets at the microscopic level.
 - Facets are small enough (not visible) and big enough (no interference & diffraction)
- The aggregate behavior of these facets determines the reflectance.
- Two important factors:
 - How individual facet reflects light?
e.g. perfect mirrors (Cook-Torrance) or perfect Lambertian (Oren-Nayar)
 - What is the distribution of facet orientations (normal distribution function)?
e.g. Gaussian distribution



Microfacet Theory

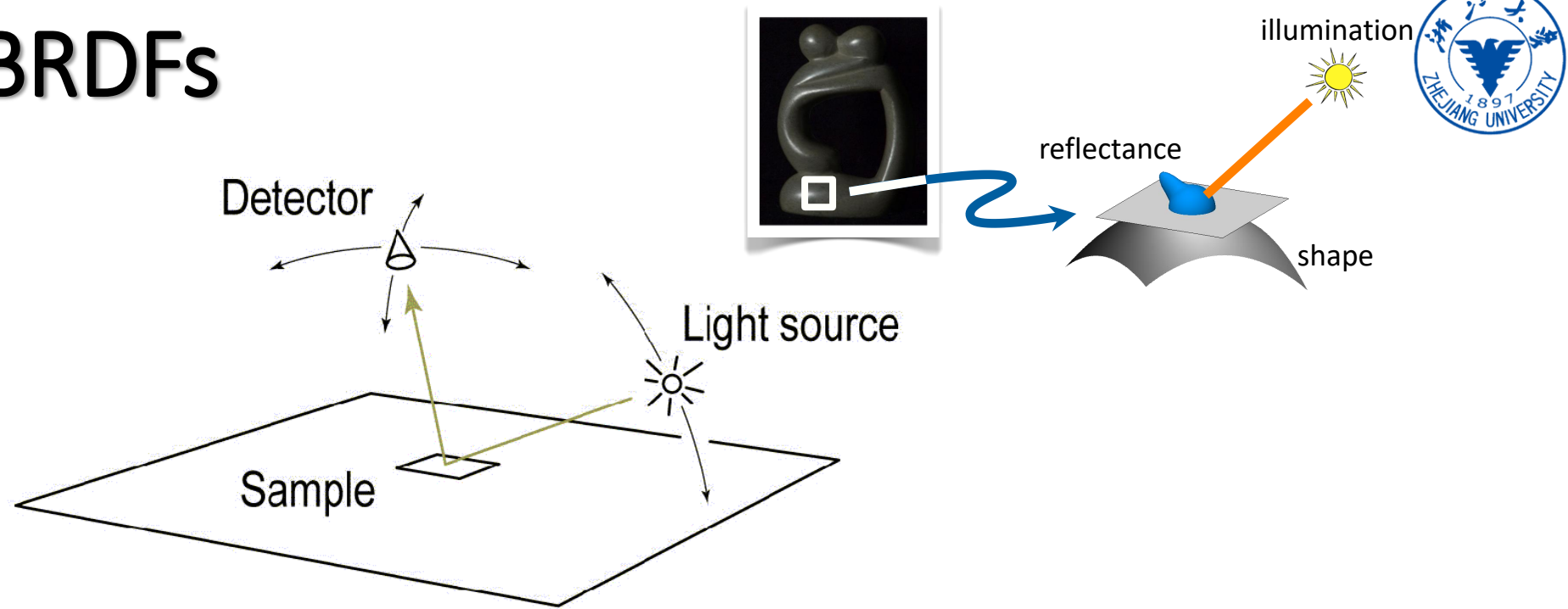
- Example: “V-grooves” on brushed metal surface



Questions?



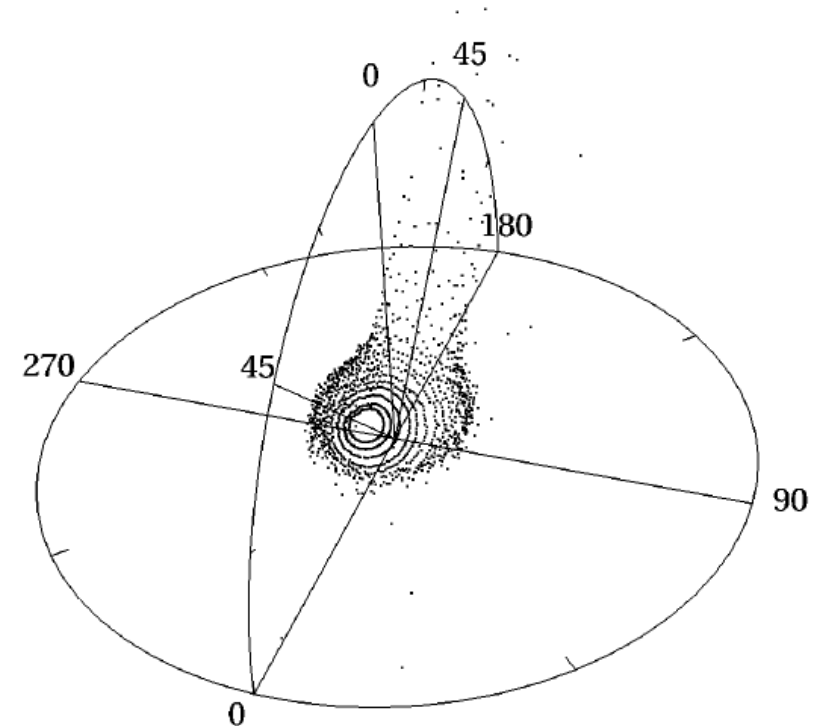
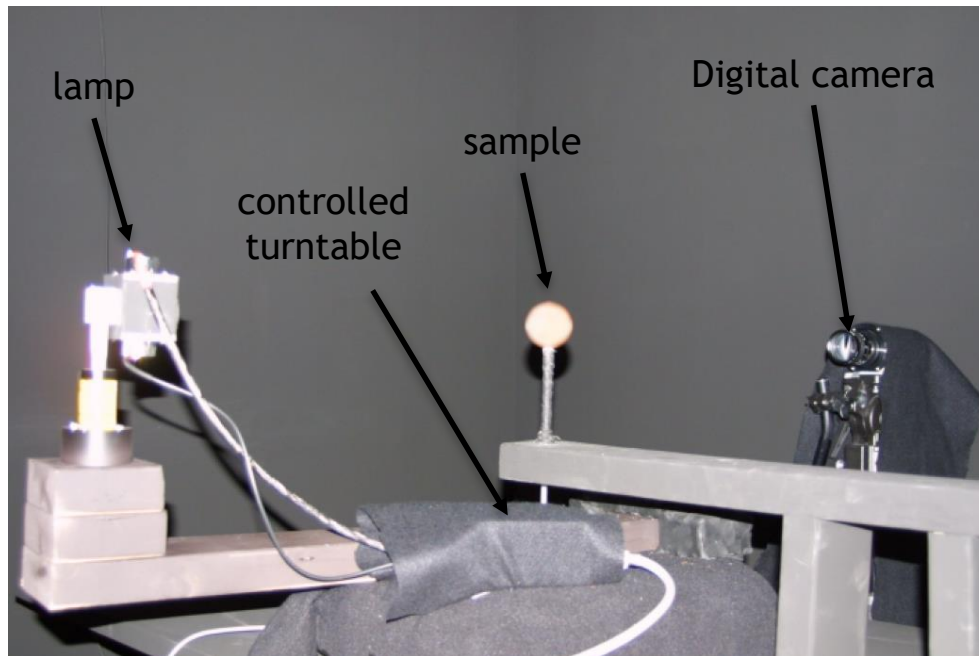
Capturing BRDFs



- Radiance (pixel intensity) is determined by shape, lighting, and BRDF
- Capture BRDF from images with known shape and lighting
- Often capture a flat sample with a moving light and camera
 - Need careful calibration of light and camera
(also a darkroom to avoid inter-reflection, e.g. from the white walls)

More Efficient BRDF Capture

- Use a homogeneous spherical sample of the material
 - A sphere (known shape) contains all kinds of normals
 - So a single image contains many BRDF samples
 - Still need to move the light or camera

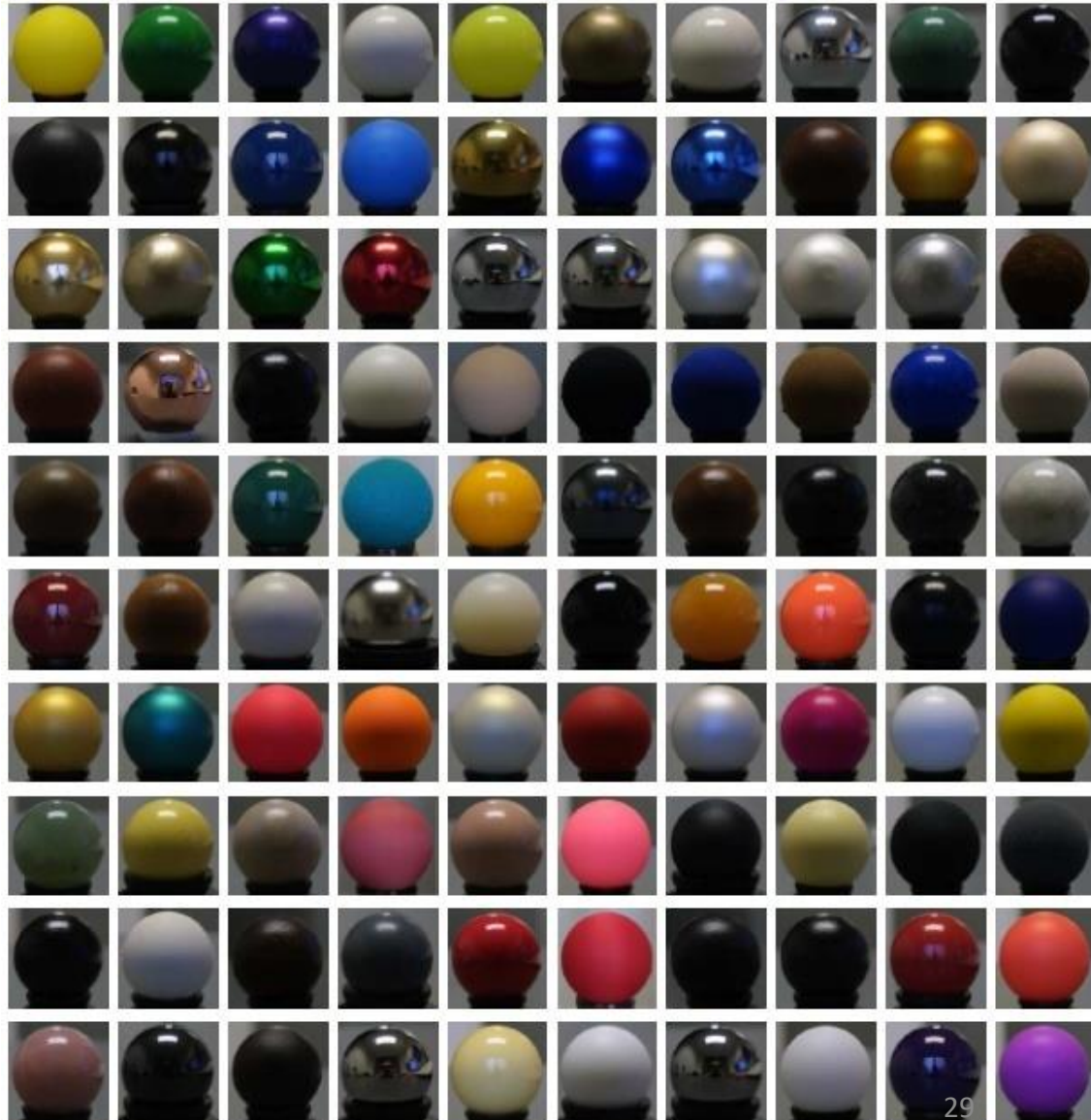




Represent Captured BRDFs

- Tabulated BRDF
 - 4D table $\rho(\omega_i, \omega_o) = \rho(\theta_i, \phi_i, \theta_o, \phi_o)$
 - Not editable
- Measure-then-fit analytic models
 - Fitting can reduce noise but also is limited by the model
 - Non-obvious error metric for fitting – often biased to specular which has large values
 - Difficult optimization – nonlinear; depends on initial guess

Acquisition



130 materials were scanned;
100 of them shown here

MERL BRDF database, freely
available online

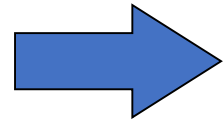
Tabulated BRDF



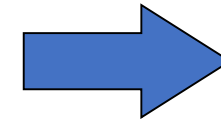
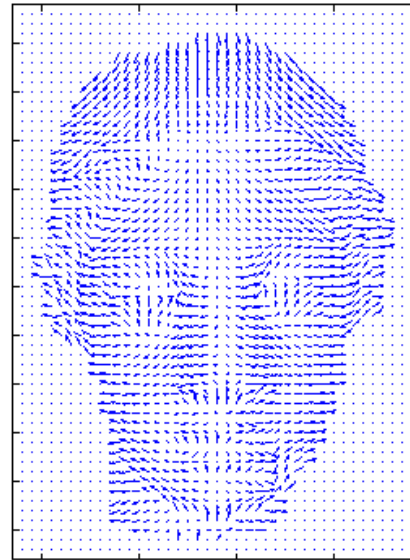
Questions?



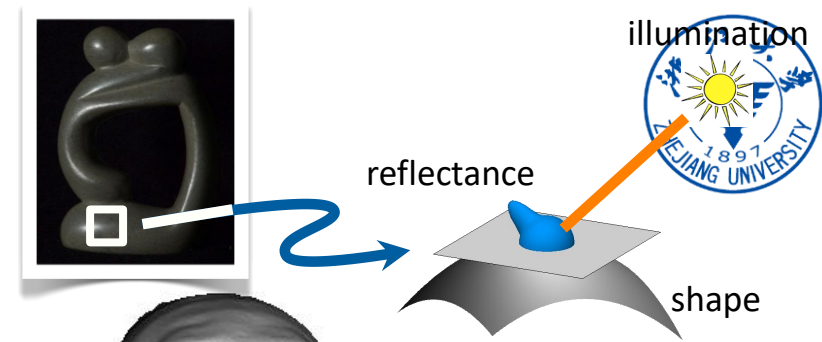
Photometric Stereo (Capturing Shapes)



Photometric
stereo

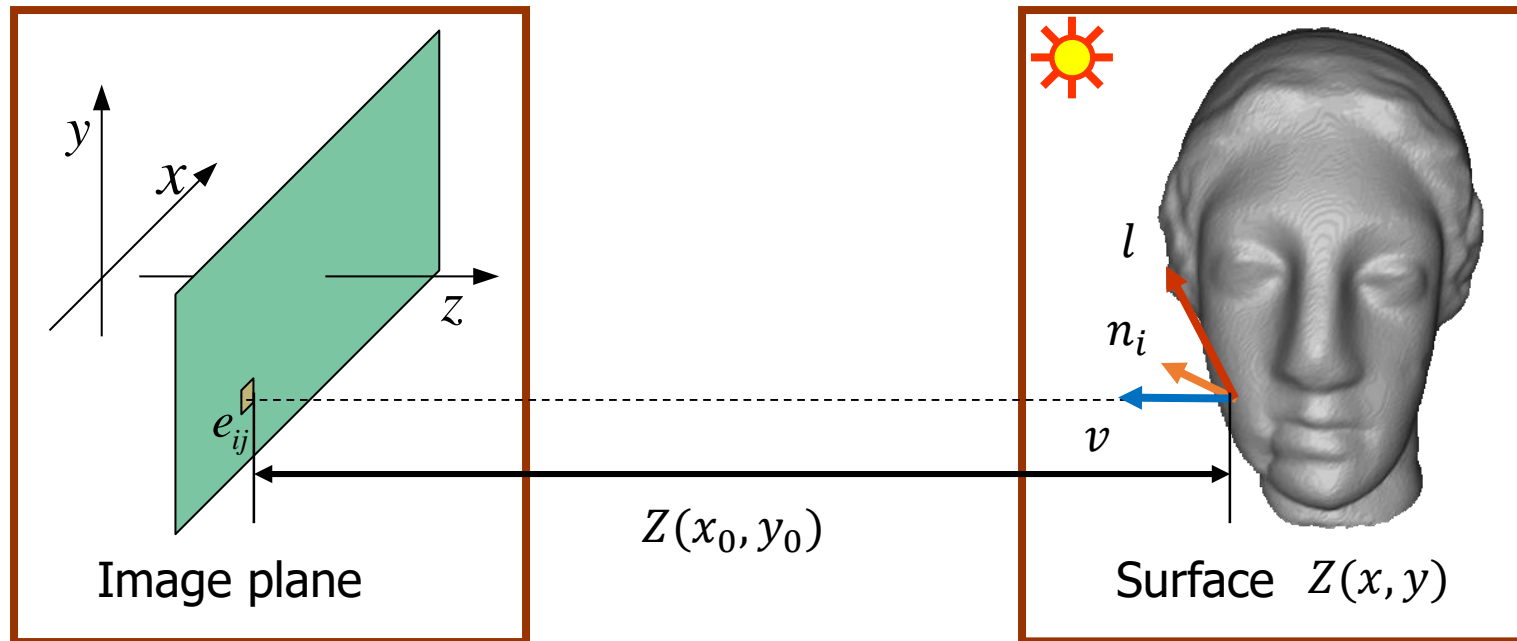


Integration



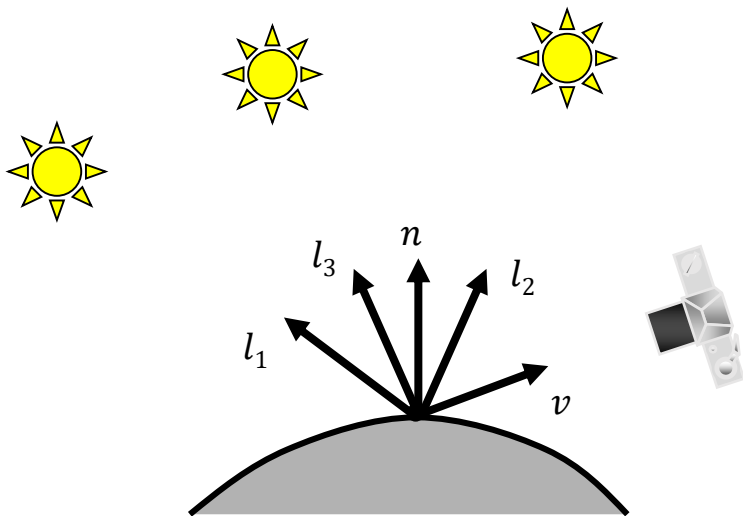
- Radiance (pixel intensity) is determined by shape, lighting, and BRDF
- Capture shape from images with known BRDF and lighting
- Often captured with a fixed camera and a moving light
 - Often assume Lambert's reflectance (pure diffuse material)
 - Need careful calibration of lighting
(also a darkroom to avoid inter-reflection, e.g. from the white walls)

Typical Assumptions



- Lambert's reflectance model
- Camera centered coordinate system
- Orthographic camera (v is the same for all pixels)
- Directional illumination (l is the same for all pixels)

Lambertian Photometric Stereo



$$I_1 = \rho \mathbf{n} \cdot \mathbf{l}_1$$


$$I_2 = \rho \mathbf{n} \cdot \mathbf{l}_2$$

$$I_3 = \rho \mathbf{n} \cdot \mathbf{l}_3$$

By the Lambert's reflectance model

Write in a matrix equation:

$$\underbrace{\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}}_{\mathbf{I} \quad 3 \times 1} = \underbrace{\begin{pmatrix} \mathbf{l}_1^T \\ \mathbf{l}_2^T \\ \mathbf{l}_3^T \end{pmatrix}}_{\mathbf{L} \quad 3 \times 3} \underbrace{\rho \mathbf{n}}_{\mathbf{b} \quad 3 \times 1}$$

L is known 

$$\mathbf{b} = \mathbf{L}^{-1} \mathbf{I}$$

$$\rho = \|\mathbf{b}\|$$

$$\mathbf{n} = \frac{1}{\rho} \mathbf{b}$$





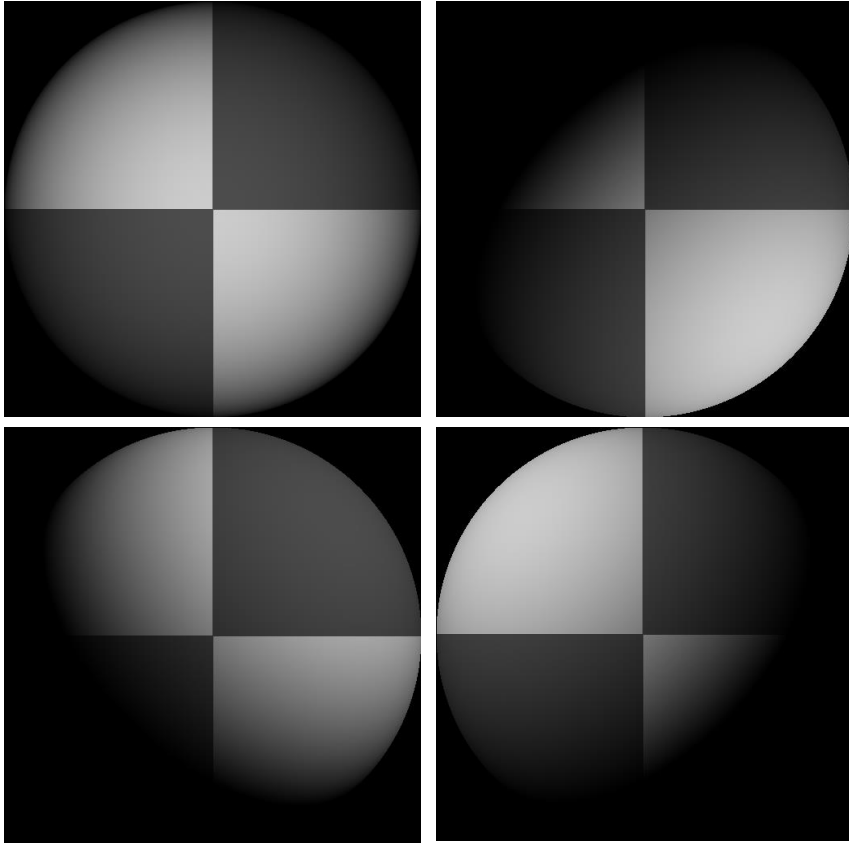
Dealing with Shadows

- The linear formulation, $I = \rho \mathbf{n} \cdot \mathbf{l}$, does not consider shadows
- Pixels in “attached shadows” have zero intensity, $I = 0$, while $\mathbf{n} \cdot \mathbf{l} < 0$
- A better formulation is nonlinear:

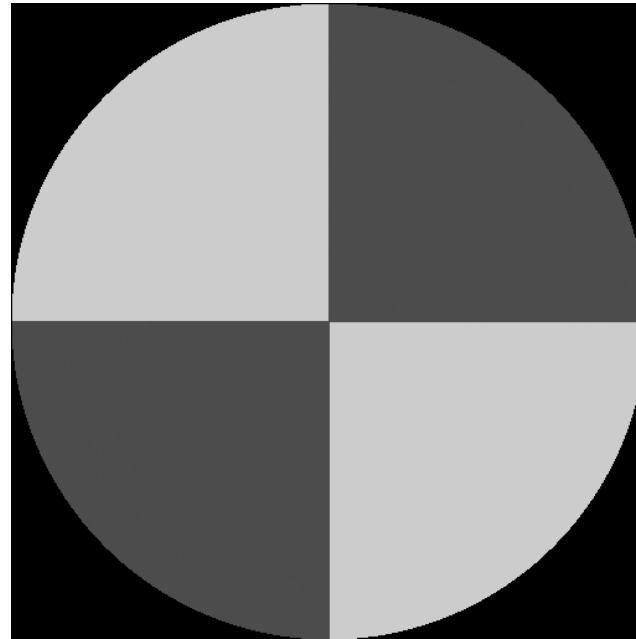
$$I = \max(0, \rho \mathbf{n} \cdot \mathbf{l})$$

- But $\max(0, \cdot)$ is a nonlinear function, difficult to fit
- A simple way to deal with shadows and keep linear formulation
 - At each pixel, there are multiple observations I_1, I_2, \dots, I_K
 - Suppose K is large and the variation in lighting directions is large
 - So there are enough observations in I_1, I_2, \dots, I_K that are free from shadows
 - Thus, we can sort I_1, I_2, \dots, I_K by their values and discard the 20% darkest

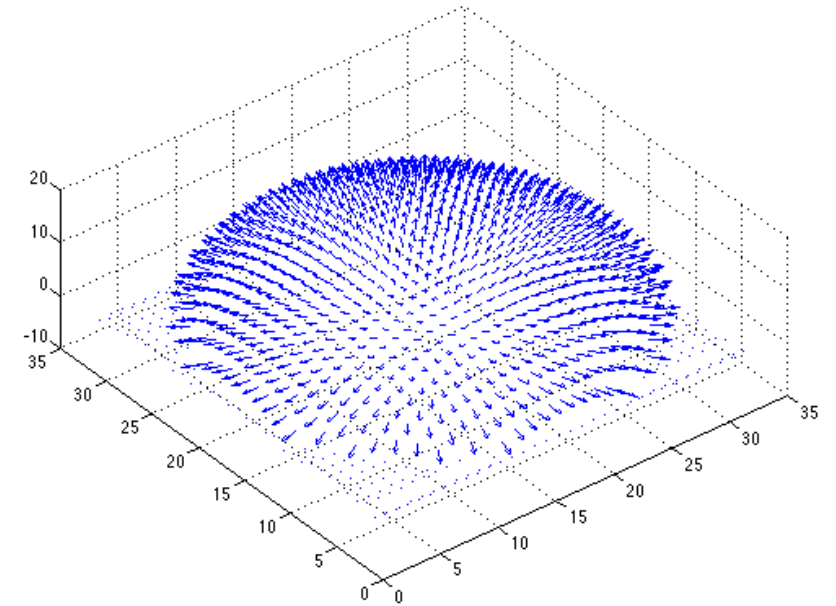
Example Figures



Input images



Albedo ρ



Normal

Questions?



Depth from Normals (Method I)

- Suppose the surface is $(x, y, Z(x, y))$
- The surface normal then should be,

$$\mathbf{n}(x, y) = \frac{1}{\sqrt{Z_x^2 + Z_y^2 + 1}} \begin{pmatrix} -Z_x \\ -Z_y \\ 1 \end{pmatrix}$$

- If we denote the normal as,

$$\mathbf{n}(x, y) = \begin{pmatrix} n_1(x, y) \\ n_2(x, y) \\ n_3(x, y) \end{pmatrix}$$

- Then we obtain the following partial derivatives:

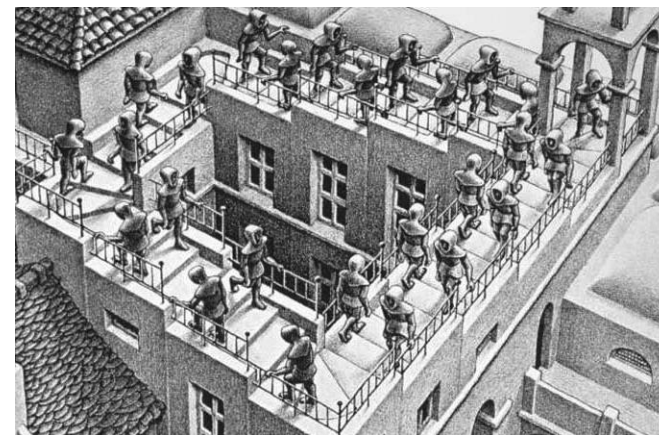
$$Z_x(x, y) = -n_1(x, y)/n_3(x, y)$$

$$Z_y(x, y) = -n_2(x, y)/n_3(x, y)$$

Depth from Normals (Method I)

- We can now recover the surface height at any point by integration along some path, e.g.

$$Z(x, y) = \int_0^x Z_x(s, y) ds + \int_0^y Z_y(x, t) dt + c$$



- This method never works on real data. Why?
- The recovered normal is too noisy!
- Recall that mixed second partials are equal --- this gives us a **check**. We must have:

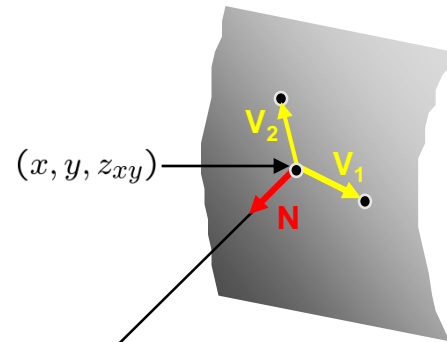
$$\frac{\partial Z_x(x, y)}{\partial y} = \frac{\partial Z_y(x, y)}{\partial x}$$

(or they should be similar, at least)

- Due to imaging and estimation noise, this almost never happens.

Depth from Normals (Method II)

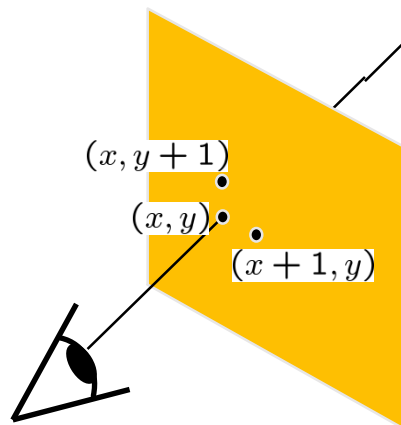
- The tangent vector \mathbf{v}_1 is perpendicular to \mathbf{n}



$$\begin{aligned}\mathbf{v}_1 &= (x + 1, y, Z(x + 1, y)) - (x, y, Z(x, y)) \\ &= (1, 0, Z(x + 1, y) - Z(x, y))\end{aligned}$$

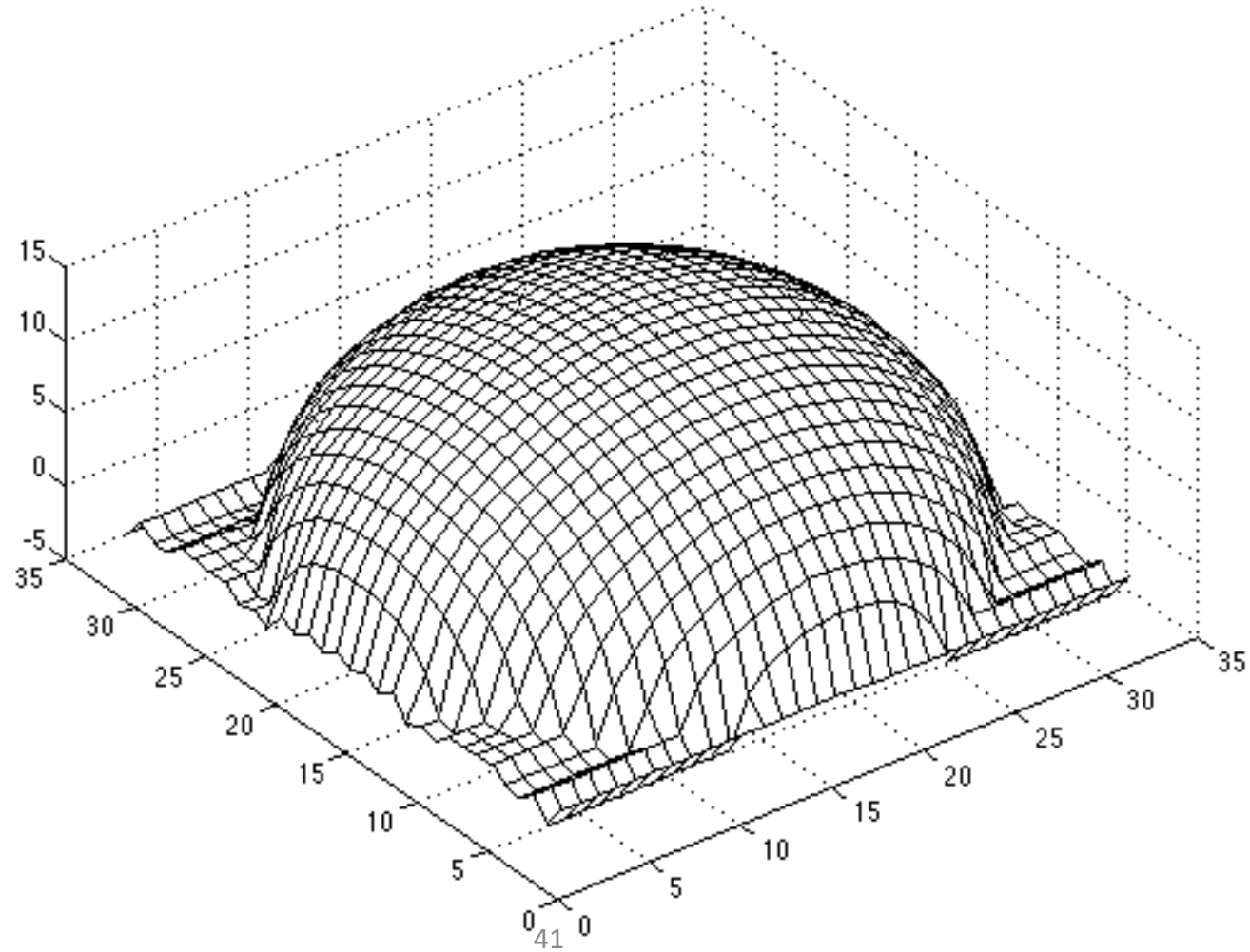
$$\begin{aligned}0 &= \mathbf{n} \cdot \mathbf{v}_1 \\ &= (n_1, n_2, n_3) \cdot (1, 0, Z(x + 1, y) - Z(x, y)) \\ &= n_1 + n_3(Z(x + 1, y) - Z(x, y))\end{aligned}$$

- Get a similar equation for \mathbf{v}_2
- Each normal gives two linear constraints on Z
- Compute Z values by solving a matrix equation



This leads to a large sparse linear equation.
Often solved by the Conjugated Gradient algorithm.

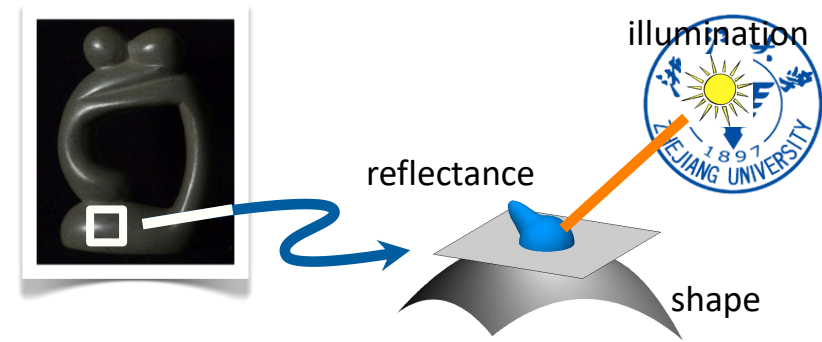
Surface Recovered



Questions?



Capturing Lighting



- Radiance (pixel intensity) is determined by shape, lighting, and BRDF
- Capture lighting from images with known BRDF and shape
- Often captured with mirror spheres
 - Known shape (sphere) and known BRDF (mirror)

Capture a Directional Light

- For example, to get the matrix L in photometric stereo (page 34)
- Capture a shiny sphere in the scene
 - the location of the highlight tells the lighting direction



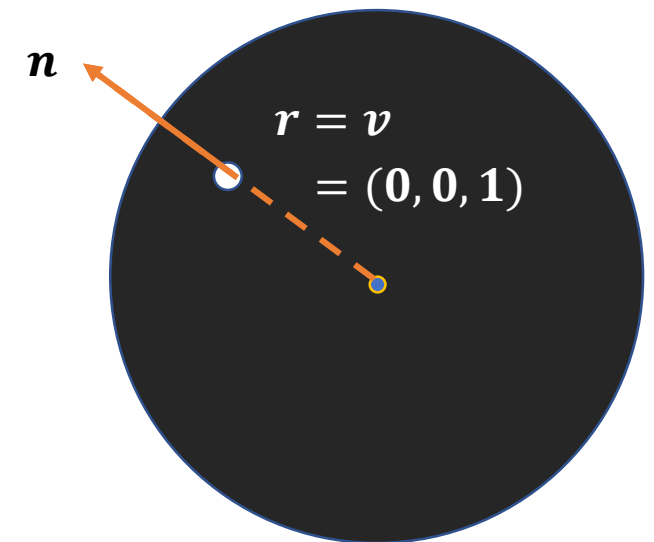
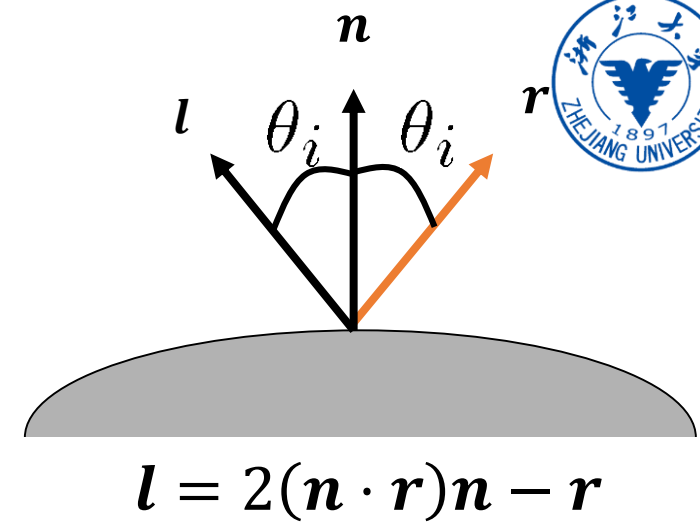
Capture a Directional Light



- For a mirror sphere, light is reflected about \mathbf{n}

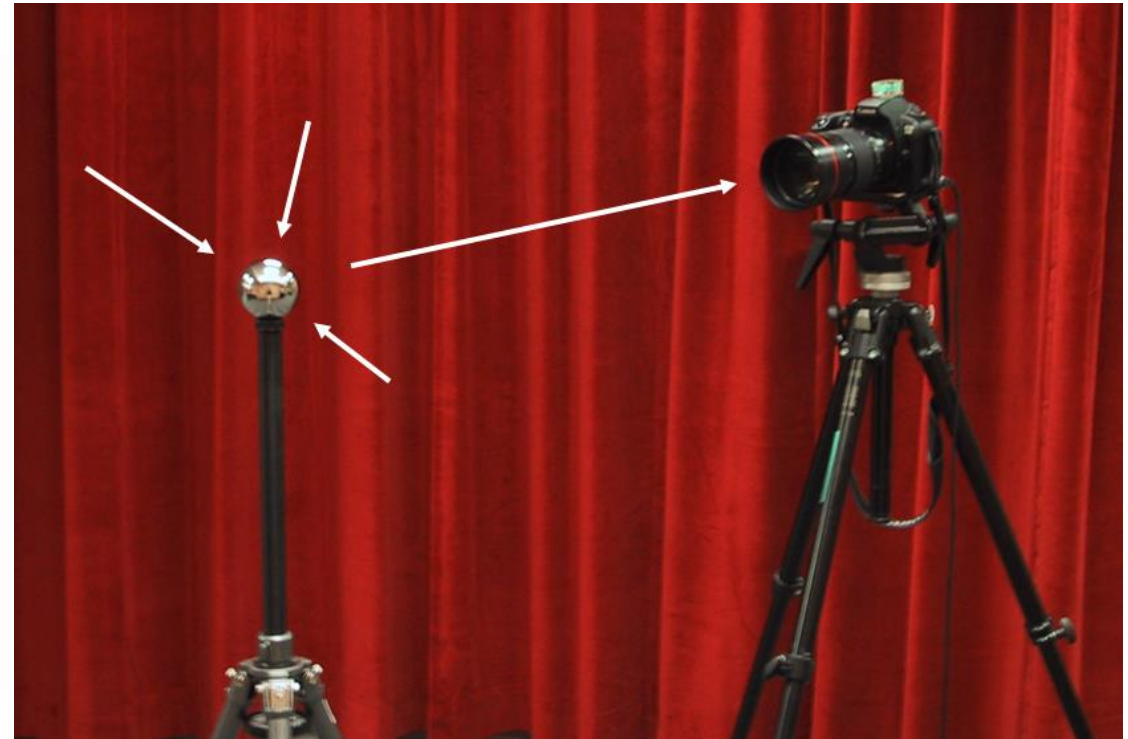
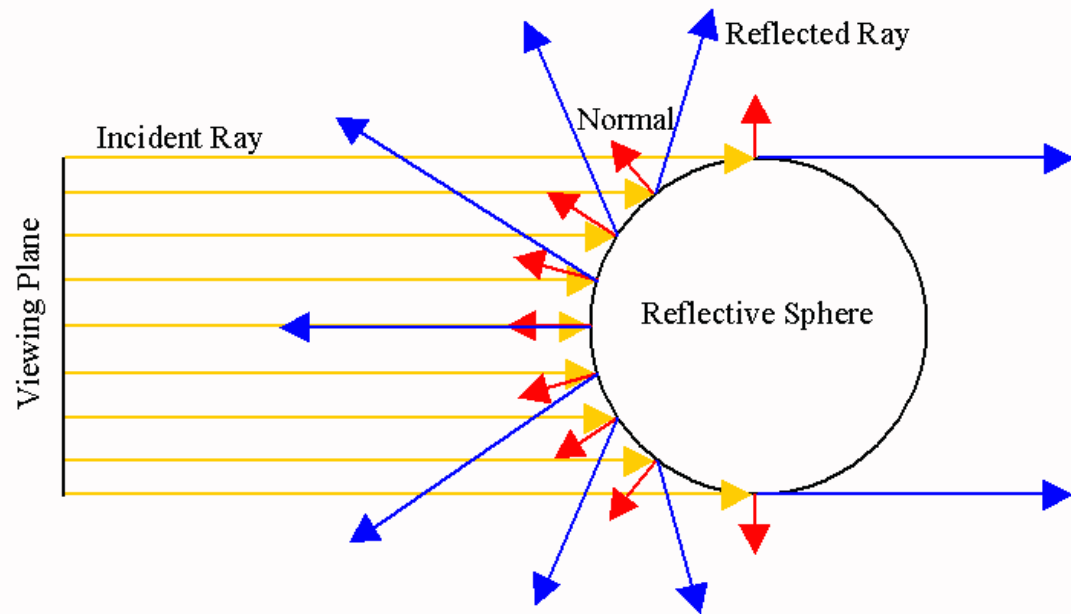
$$L_o = \begin{cases} L_i & \text{if } \mathbf{v} = \mathbf{r} \\ 0 & \text{otherwise} \end{cases}$$

- The light source is seen at a pixel where $\mathbf{v} = \mathbf{r}$
- Assume orthographic camera
 - $\mathbf{v} = (0,0,1)$ for all pixels
- So if we further know \mathbf{n} , we can compute \mathbf{l}
 - normal of each point on a sphere can be determined
 - (by some simple geometry, try to derive it yourself)



Capture an Environment Light

- Trace back all light rays reflected from a mirror sphere
- Still assume orthographic camera



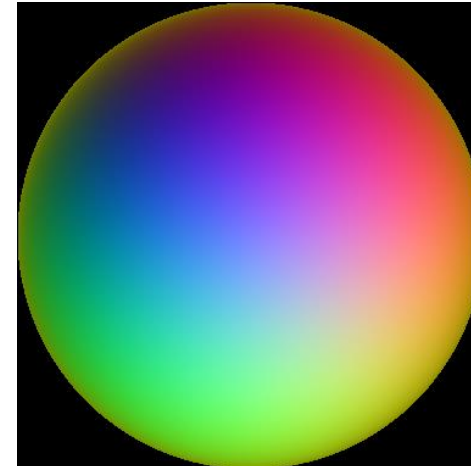
Mirror ball -> equirectangular



Mirror ball



Normals



Reflection vectors



Equirectangular (longitude & latitude)

Applications of Captured Lighting

- Rendering virtual objects into real scenes (e.g. in AR)

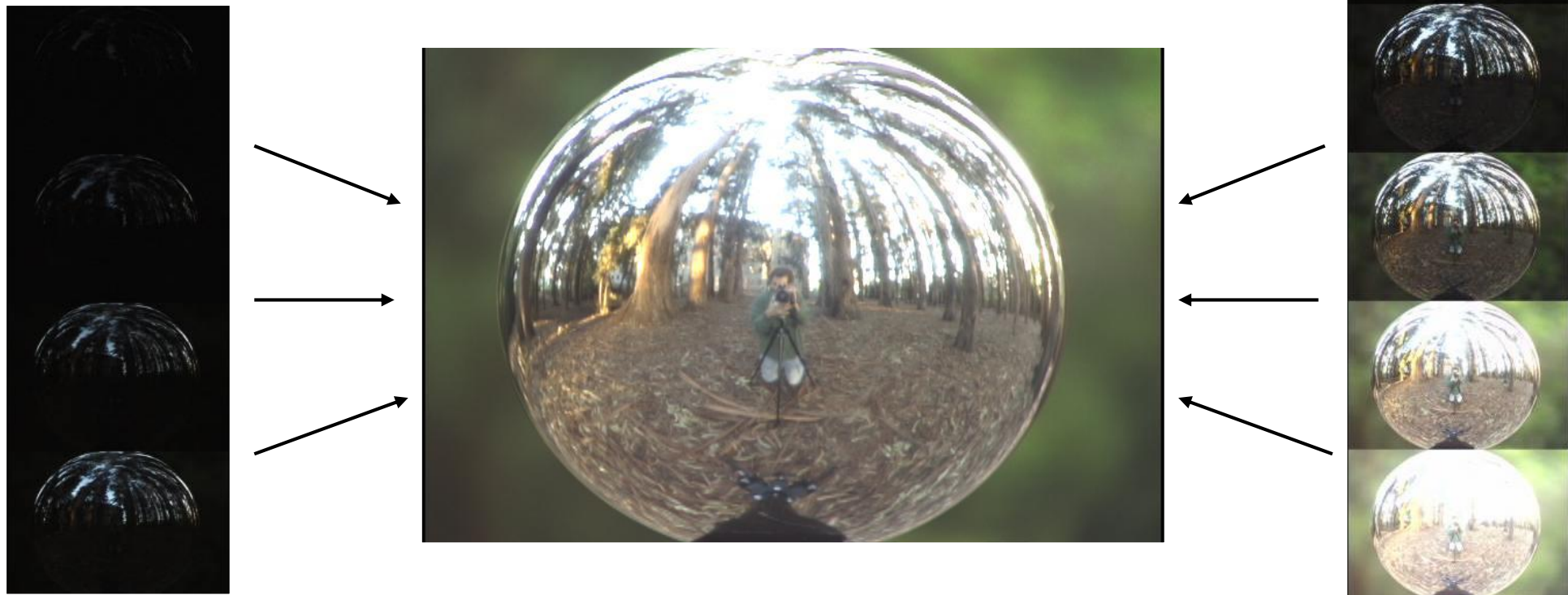




from Terminator 2

HDR Environment Lighting

- HDR is needed so that light probes capture full range of radiance



Real-World HDR Lighting Environments

Funston
Beach



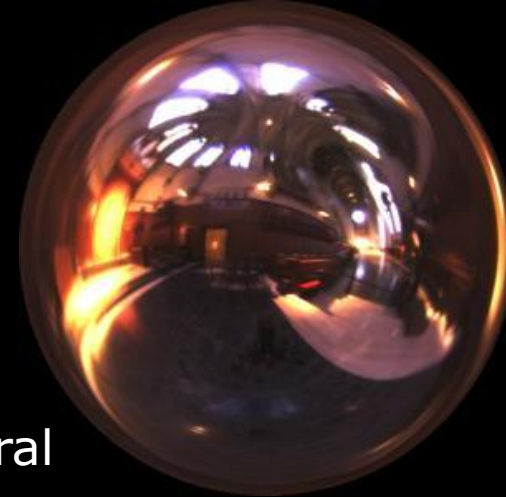
Eucalyptus
Grove

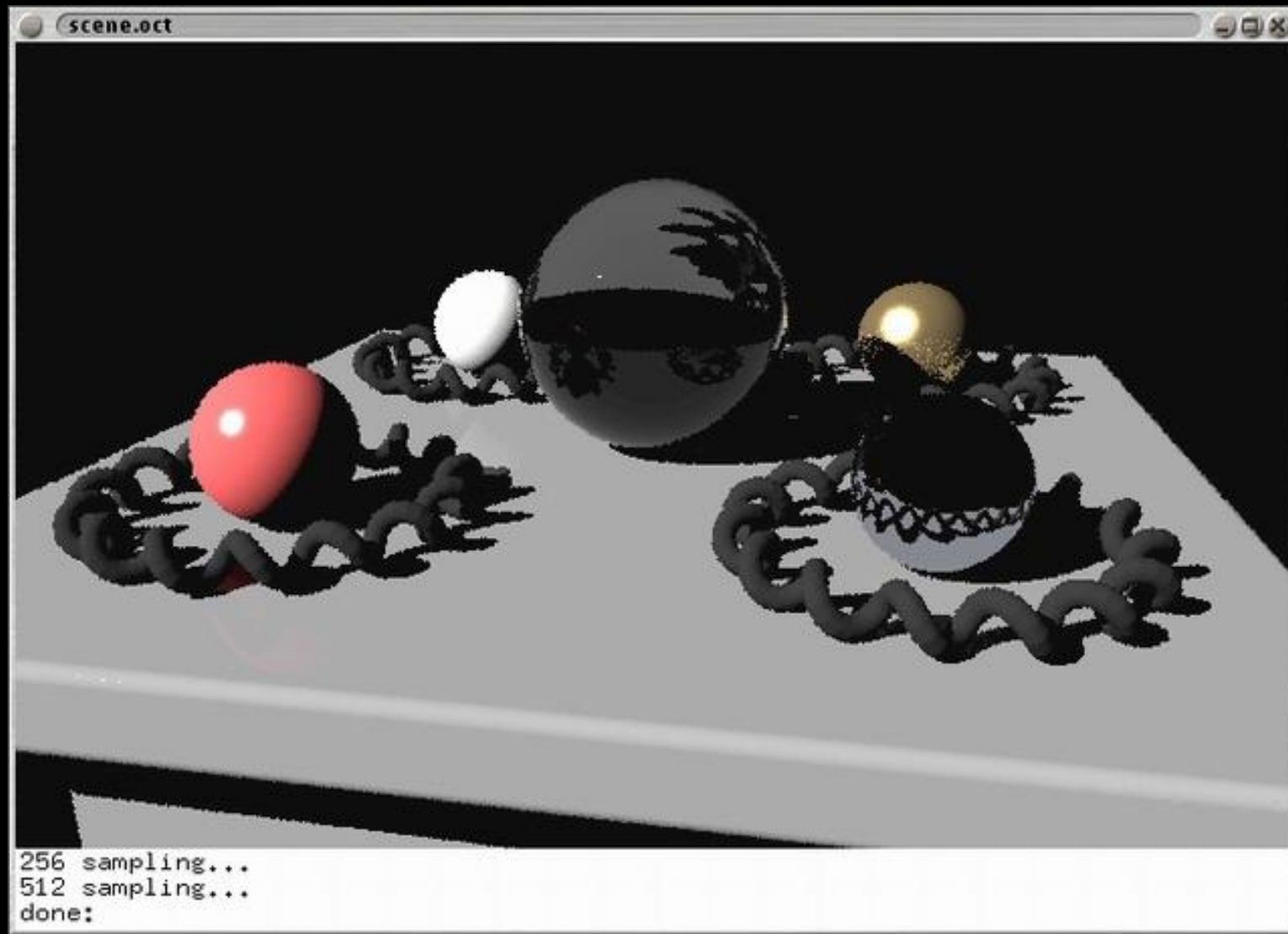


Uffizi
Gallery



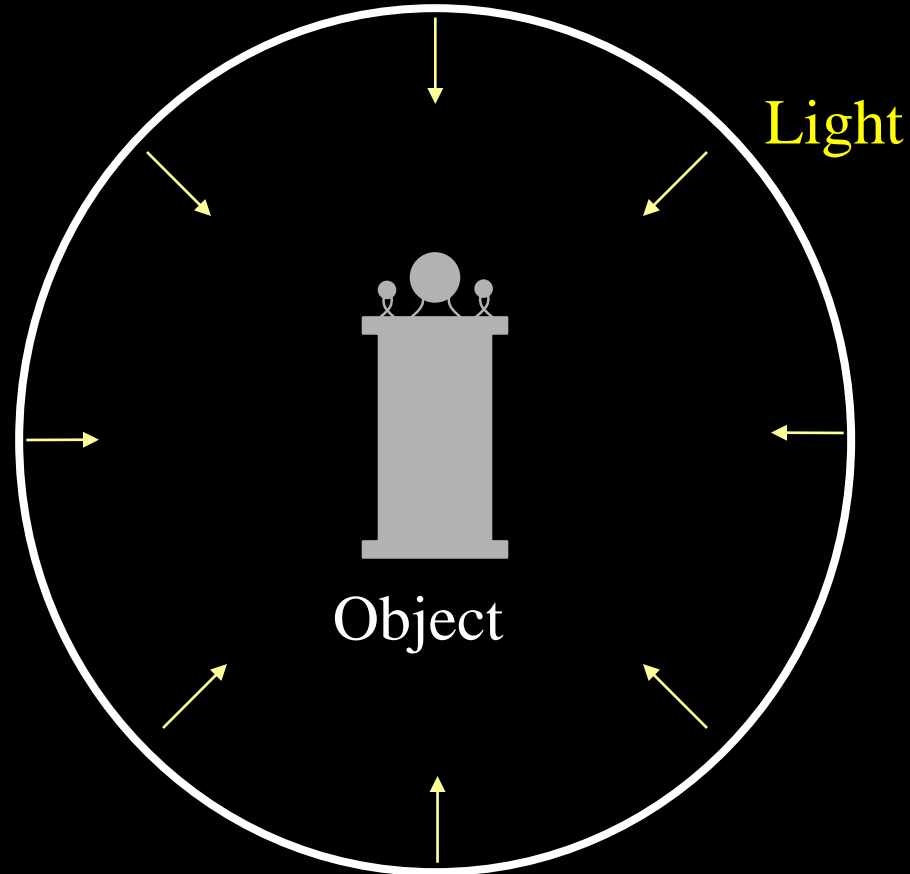
Grace
Cathedral





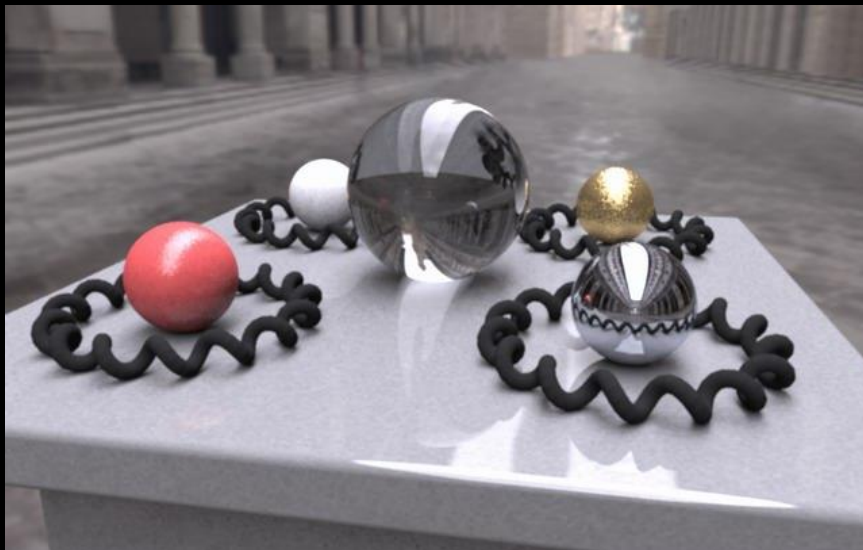
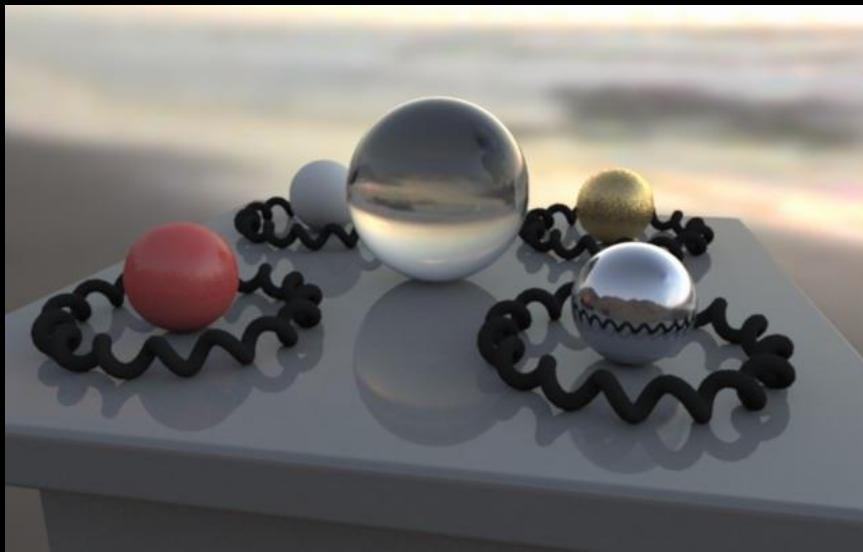
CG Objects Illuminated by a Traditional CG
Light Source

Illuminating Objects using Measurements of Real Light



Environment
assigned "glow"
material
property in
Greg Ward's
RADIANCE
system.

<http://radsite.lbl.gov/radiance/>



Paul Debevec. A Tutorial on Image-Based Lighting. IEEE Computer Graphics and Applications, Jan/Feb 2002.

Questions?





Homework 1: Photometric Stereo

- Deadline: **2025.12.07 23:59**

Homework 1: Photometric Stereo

取消发布

编辑

作业属性

作业批改

成绩统计

活动时间 结束于2026.01.31 23:59

占成绩比例 15.00%

公布成绩时间 不公布

作业形式 个人作业 (全部学生: 99人)

计分规则 最后一次给分

完成指标 提交作业

评分方式(教师评阅 100.0%)

选择量规

教师评阅

占成绩比例 100.0%

量规

作业说明

Please refer to the supplementary files (附件) for the detailed illustration of the homework (Homework 1.pdf) and the test data (Homework 1.zip).

Note that the deadline for the submission is 23:59, December 07th (2025). You can still upload it after ddl, but the penalty of 20% score reduction per 12 hours after ddl will be applied.



Submission & Late Policy

- Assignments are submitted via Learning in ZJU (学在浙大) .



- For a late submission, penalty of 20% score reduction per 12 hours is applied. If one submits an assignment within 12 hours after the deadline, the score becomes 80%. Within 24 hours, 60%, and so on.